The Introduction To Artificial Intelligence

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The Introduction to Artificial Intelligence

- Part I Brief Introduction to AI & Different AI tribes
- Part II Knowledge Representation & Reasoning

- **Brief Review**

1.3 Knowledge Representation & Reasoning

What is propositional logic

- A *proposition* is a declarative sentence that is either true or false.
- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Beijing is the capital of China.
  c) Hangzhou is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions.
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

1.3 Knowledge Representation & Reasoning

What is propositional logic

- Constructing Propositions
  - Propositional Variables: p, q, r, s, …
  - The proposition that is always true is denoted by **T** and the proposition that is always false is denoted by **F**.
  - Compound Propositions: constructed from logical connectives and other propositions
    - Negation $\neg$
    - Conjunction $\wedge$
    - Disjunction $\vee$
    - Implication $\longrightarrow$
    - Biconditional $\leftrightarrow$

1.3 Knowledge Representation & Reasoning

Syntax of logical connectives

Conjunction	: And	$\wedge$	
Disjunction	: Or	$\vee$	
Negation	: Not	$\neg$	
Implication	: Implies	$\rightarrow$	(if… then…)
Biconditional	: Iff	↔️	(if and only if)

1.3 Knowledge Representation & Reasoning

- What is propositional logic

A	B	¬ A	A ∧ B	A ∨ B	A → B	A ↔ B
t	t	f	t	t	t	t
t	f	f	f	t	f	f
f	t	t	f	t	t	f
f	f	t	f	f	t	t

1.3 Knowledge Representation & Reasoning

Tautologies, Contradictions, and Contingencies

- A tautology is a proposition which is always true.
  - Example: $p \vee \neg p$
- A *contradiction* is a proposition which is always false.
  - Example: $p \wedge \neg p$
- A *contingency* is a proposition which is neither a tautology nor a contradiction, such as p

1.3 Knowledge Representation & Reasoning

Logical Equivalences

- Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.
- We write this as $p \Leftrightarrow q$ or as $p \equiv q$ where p and q are compound propositions.
- Two compound propositions p and q are equivalent if and only if the columns in a truth table giving their truth values agree.
- This truth table shows that $\neg p \vee q$ is equivalent to $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

1.3 Knowledge Representation & Reasoning

De Morgan's Laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

Augustus De Morgan

1806-1871

This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \lor q)$	$\neg(p \lor q)$	$\neg p \land \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

1.3 Knowledge Representation & Reasoning

Logical Equivalences

- *The operations* ∧,∨ *are commutative and associative, and the*
- *following equivalences are generally valid*:
- $\neg A \lor B \equiv A \rightarrow B$ (implication)
- $A \rightarrow B \equiv \neg B \rightarrow \neg A$ (contraposition)
- $(A \rightarrow B) \land (B \rightarrow A) \equiv (A \leftrightarrow B)$ (equivalence)
- $\neg (A \land B) \equiv \neg A \lor \neg B$ (De Morgan's law)
- $\neg (A \lor B) \equiv \neg A \land \neg B$
- $A \lor (B \land C) \equiv (A \lor B) \land (A \lor C)$ (distributive law)
- $A \land (B \lor C) \equiv (A \land B) \lor (A \land C)$
- $A \lor \neg A \equiv t$ (tautology)
- $A \land \neg A \equiv f$ (contradiction)
- $A \lor f \equiv A$
- $A \lor t \equiv t$
- $A \land f \equiv f$
- $A \land t \equiv A$

Homework-1

Logic Puzzles

Knights: t
Knaves: f

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says “At least one of us is a knave.”
  - B says nothing.

Example: What are the types of A and B?

Homework-1

Logic Puzzles

- **Solution:** Let p and q be the statements that A is a knight and B is a knight, respectively. So, then $\neg p$ represents the proposition that A is a knave and $\neg q$ that B is a knave.
  - If A is a knight, then p is true. Since knights tell the truth, q must also be true. Then, $p \wedge \neg q$ would have to be true, which is logically right. So, A is a knight and B is a knave.
  - If A is a knave, then $p \wedge q$ would have to be true, but it is not. So, A is not a knave.
  - In conclusion: A is a knight, B is a knave.

1.3 Knowledge Representation & Reasoning

- What is knowledge representation?
- Propositional Logic
- *Predicate Logic*
- Production-rule System
- Frame-Based System
- State Space System
- Knowledge graph

1.3 Knowledge Representation & Reasoning

What is propositional logic

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- Examples of propositions:
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- Examples that are not propositions.
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1.3 Knowledge Representation & Reasoning

Predicate Logic

- Consider the following statements:

$$x > 3, x = y + 3, x + y = z$$

- The truth value of these statements has no meaning without specifying the values of x, y, z
- However, we can make propositions out of such statements.
- A predicate is a property that is affirmed or denied about the subject (in logic, we say “variable” or argument) of a statement.

$$\text{“}\underbrace{x}_{\text{subject}}\ \underbrace{\text{is greater than 3”}}_{\text{predicate}}$$

Terminology: affirmed = holds = is true; denied = does not hold = is not true.

1.3 Knowledge Representation & Reasoning

Predicate Logic

- To write in predicate logic:

$$\text{“}\underbrace{x}_{\text{subject}}\ \underbrace{\text{is greater than 3”}}_{\text{predicate}}$$

- We introduce a (functional) symbol for the predicate, and put out the subject as an argument (to the functional symbol): $P(x)$
- Examples:

(1) Father(x): unary predicate

(2) Brother(x, y): binary predicate

(3) Sum(x, y, z): ternary predicate

(4) P(x, y, z, t): n-ary predicate

1.3 Knowledge Representation & Reasoning

Predicate Logic

- Definition: A statement of form $P(x_1, x_2, \cdots, x_n)$ is the value of the propositional function P. Here, $(x_1, x_2, \cdots, x_n)$ is an n-tuple and P is a predicate.

- You can think of a propositional function as a function that

  (1) Evaluates to true or false

  (2) Take one or more arguments

  (3) Expresses a predicate involving the argument(s).

  (4) Becomes a proposition when values are assigned to the arguments.

1.3 Knowledge Representation & Reasoning

Predicate Logic

- Example:

Let $Q(x, y, z)$ denote the statement "$x^2 + y^2 = z^2$". What it the truth value of $Q(3,4,5)$? What is the truth vale of $Q(2,2,3)$? How many values of (x, y, z) make the predicate true?

1.3 Knowledge Representation & Reasoning

Predicate Logic

- Example:

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- Answer:

Since $3^2 + 4^2 = 25 = 5^2$, $Q(3,4,5)$ is true.

Since $2^2 + 2^2 = 8 \neq 3^2 = 9$, $Q(2,2,3)$ is false.

There are infinitely many values for (x, y, z) that make this propositional function true --- how many right triangles are there?

1.3 Knowledge Representation & Reasoning

Predicate Logic

- Consider the previous example. Does it make sense to assign to x the value "blue"?

- Intuitively, *the universe of discourse* is the set of all things we wish to talk about; that is, the set of all objects that we can sensibly assign to variable in a propositional function.

- What would be the universe of discourse for propositional function $P(x) =$ "The test will be on x the 23rd" be ?

1.3 Knowledge Representation & Reasoning

Predicate Logic

- Moreover, each variable in an n-tuple may have a different universe of discourse.

- Let $P(r, g, b, c) =$ "The rgb-value of the color c is (r, g, b)".

- For example, $P(255{,}0{,}0, red)$ is true, while $P(0{,}0{,}255, green)$ is false.

- What are the universe of discourse for (r, g, b, c)?

1.3 Knowledge Representation & Reasoning

Quantifiers

- A predicate becomes a proposition when we assign it fixed values.

- However, another way to make a predicate into a proposition is to *quantify* it. That is, the predicate is true (or false) for *all* possible values in the universe of discourse or for *some* value(s) in the universe of discourse.

- Such *quantification* can be done with two quantifiers: the *universal* quantifier and the *existential* quantifier.

1.3 Knowledge Representation & Reasoning

Quantifiers

- Definition: The universal quantification of a predicate $P(x)$ is the proposition "$P(x)$ is true for all values of x in the universe of discourse". We use the notation

$$\forall x P(x)$$

which can be read "for all x".

- If the universe of discourse is finite, say $\{n_1, n_2, \cdots, n_k\}$, then the universal quantifier is simply the conjunction of all elements:

$$\forall x P(x) \Leftrightarrow P(n_1) \wedge P(n_2) \wedge P(n_3) \wedge \cdots \wedge P(n_k)$$

1.3 Knowledge Representation & Reasoning

Quantifiers --- Example 1

- Let $P(x)$ be the predicate "x must take discrete mathematics course" and let $Q(x)$ be the predicate "x is a computer science student."
- The universe of discourse for both $P(x)$ and $Q(x)$ is all students.
- Express the statement "Every computer science student must take discrete mathematics course".

$$\forall x(Q(x) \rightarrow P(x))$$

- Express the statement "Everybody must take a discrete mathematics course or be a computer science student"

$$\forall x(Q(x) \vee P(x))$$

1.3 Knowledge Representation & Reasoning

Quantifiers --- Example 2

- Express the statement "for every x and for every y, $x + y > 10$"

1.3 Knowledge Representation & Reasoning

Quantifiers --- Example 2

- Express the statement "for every x and for every y, $x + y > 10$"

- Let $P(x, y)$ be the statement $x + y > 10$ where the universe of discourse for x, y is the set of integers.

- Answer:

$$\forall x \forall y P(x, y)$$

- Or

$$\forall x, y P(x, y)$$

1.3 Knowledge Representation & Reasoning

Quantifiers

- Definition: The *existential quantification* of a predicate $P(x)$ is the proposition "There exist an x in the universe of discourse such $P(x)$ is true". We use the notation

$$\exists x P(x)$$

which can be read "there exist an x".

- Again, If the universe of discourse is finite, say $\{n_1, n_2, \cdots, n_k\}$, then the *existential* quantifier is simply the disjunction of all elements:

$$\exists x P(x) \Leftrightarrow P(n_1) \vee P(n_2) \vee P(n_3) \vee \cdots \vee P(n_k)$$

1.3 Knowledge Representation & Reasoning

Quantifiers. ---- Example 1

- Let $P(x)$ be the statement "x +y =5".

- What does the expression,

$$\exists x \exists y P(x)$$

Mean?

1.3 Knowledge Representation & Reasoning

- Quantifiers. --- Example 2
  - Express the statement "there exists a real solution to $ax^2 + bx - c = 0$"

1.3 Knowledge Representation & Reasoning

Quantifiers

- Express the statement "there exists a real solution to $ax^2 + bx - c = 0$"

- Let $P(x)$ be the statement $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where the universe of discourse for x is the set of reals. Note here that, a, b, c are all fixed constants.

- The statement can thus be expressed as

$$\exists x P(x)$$

1.3 Knowledge Representation & Reasoning

Quantifiers

- Question: what is the truth value of $\exists x P(x)$?

- Answer:
- It is false. For any real numbers such that $b^2 < 4ac$, there will only be complex solutions, for these cases no such *real* number x can satisfy the predicate.

1.3 Knowledge Representation & Reasoning

Quantifiers

- In general, when are quantified statements true/false?

Statement	True When	False When
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Table: Truth Values of Quantifiers

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers

- Existential and universal quantifiers can be used together to quantify a predicate statement; for example,

$$\forall x \exists y P(x, y)$$

is perfectly valid.

- $\forall$x $\forall$y is the same as $\forall$y $\forall$x
- $\exists$x $\exists$y is the same as $\exists$y $\exists$x

- However, you must be careful – it must be read left to right.
- For example, $\forall x \exists y P(x, y)$ is not equivalent to $\exists y \forall x(x, y)$.
- Thus, ordering is important.

1.3 Knowledge Representation & Reasoning

- Mixing Quantifiers

  For example:

  - $\exists x \, \forall y \; Loves(x, y)$
    - "Someone loves everyone."
    - "There is a person who loves everyone in the world"

  - $\forall y \, \exists x \; Loves(x, y)$
    - " everyone is loved by someone."
    - "Everyone in the world is loved by at least one person"

1.3 Knowledge Representation & Reasoning

Statement	True When	False When
$\forall x \forall y P(x, y)$	$P(x, y)$ is true for every-pair x, y.	There is at least one pair, x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x, there is a y for which $P(x, y)$ is true.	There is an x for which $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x, there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$	There is at least one pair, x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every-pair x, y.

Table: Truth Values of 2-variate Quantifiers

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers -- Example

- Express, in predicate logic, the statement that there are an infinite number of integers.

- Let $P(x, y)$ be the statement that $x < y$. Let the universe of discourse be the integers, $\mathbb{Z}$.

- Then the statement can be expressed by the following.

$$\forall x \exists y P(x, y)$$

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers -- Example

- Express *the commutative law of addition* for $\mathbb{R}$.

- We want to express that for every pair of reals, x, y the following identity holds:

$$x + y = y + x$$

- Then we have the following:

$$\forall x \forall y (x + y = y + x)$$

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers -- Example

- Express *the multiplicative inverse law for (nonzero)* rational $\mathbb{R}\backslash\{0\}$.

- We want to express that for every real number x, there exists a real number y such that $xy = 1$.

- Then we have the following:

$$\forall x \exists y (xy = 1)$$

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers -- Example

- Is commutatively for subtraction valid over the reals?

- That is, for all pairs of real numbers x, y does the identity $x - y = y - x$ hold? Express this using quantifiers.

- The expression is

$$\forall x \forall y (x - y = y - x)$$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan

1806-1871

This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers

- Just as we can use negation with proposition, we can use them with quantified expressions.
- Let $P(x)$ be a predicate. Then the following hold.

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

- This is essentially a quantified version of De Morgan's Law (in fact if the universe of discourse if finite, it is exactly De Morgan's law)
- $\forall x P(x) \Leftrightarrow P(n_1) \wedge P(n_2) \wedge P(n_3) \wedge \cdots \wedge P(n_k)$
- $\exists x P(x) \Leftrightarrow P(n_1) \vee P(n_2) \vee P(n_3) \vee \cdots \vee P(n_k)$

1.3 Knowledge Representation & Reasoning

Mixing Quantifiers -Negation

Statement	True When	False When
$\neg\exists x P(x) \equiv \forall x \neg P(x)$	For every $x, P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x) \equiv \exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Table: Truth Values of Negation Quantifiers

Homework-2

Mixing Quantifiers

- Express the statement “there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x ” as a logical expression.

- Solution:

1.3 Knowledge Representation & Reasoning

- Applications: Translation from English to logic

1. "A is above C, D is on E and above F."
2. "A is green while C is not."
3. "Everything is on something."
4. "Everything that is free has nothing on it."
5. "Everything that is green is free."
6. "There is something that is red and is not free."
7. "Everything that is not green and is above B, is red."

1.3 Knowledge Representation & Reasoning

- Translation from English to logic

1. "A is above C, D is on E and above F."
   $Above(A, C) \land Above(D, F) \land On(D, E)$

2. "A is green while C is not."
   $Green(A) \land \neg Green(C)$

3. "Everything is on something."
   $\forall x \exists y \; On(x, y)$

4. "Everything that is free has nothing on it.“
   $\forall x \; (Free(x) \rightarrow \neg \exists y, On(y, x))$

1.3 Knowledge Representation & Reasoning

- Translation from English to logic

5. "Everything that is green is free."
   $\forall x(Green(x) \rightarrow Free(x))$

6. "There is something that is red and is not free."
   $\exists x\ (Red(x) \land \neg Free(x))$

7. "Everything that is not green and is above B, is red.“
   $\forall x\ (\neg Green(x) \land Above(x, B) \rightarrow Red(x))$

1.3 Knowledge Representation & Reasoning

Translation from English to logic

Formula	Description
$\forall x \; frog(x) \Rightarrow green(x)$	All frogs are green
$\forall x \; frog(x) \wedge brown(x) \Rightarrow big(x)$	All brown frogs are big
$\forall x \; likes(x, cake)$	Everyone likes cake
$\neg\forall x \; likes(x, cake)$	Not everyone likes cake
$\neg\exists x \; likes(x, cake)$	No one likes cake
$\exists x \; \forall y \; likes(y, x)$	There is something that everyone likes
$\exists x \; \forall y \; likes(x, y)$	There is someone who likes everything
$\forall x \; \exists y \; likes(y, x)$	Everything is loved by someone
$\forall x \; \exists y \; likes(x, y)$	Everyone likes something
$\forall x \; customer(x) \Rightarrow likes(bob, x)$	Bob likes every customer
$\exists x \; customer(x) \wedge likes(x, bob)$	There is a customer whom bob likes
$\exists x \; baker(x) \wedge \forall y \; customer(y) \Rightarrow mag(x, y)$	There is a baker who likes all of his customers

Homework-3

- Rewrite the expression

$$\neg\forall x(\exists y\forall x P(x,y,z)\wedge\exists z\forall y P(x,y,z))$$

- Let $P(x,y)$ denote "x is a factor of y" where $x\in\{1,2,3,\cdots\}$ and y $\in\{1,2,3,\cdots\}$. Let $Q(y)$ denote "$\forall x[P(x,y)\rightarrow((x=y)\vee(x=1))]$" . When is $Q(y)$ true?