The Introduction To Artificial Intelligence

Yuni Zeng yunizeng@zstu.edu.cn

2024-2025-1

The Introduction to Artificial Intelligence

- Part I Brief Introduction to AI & Different AI tribes
- Part II Knowledge Representation & Reasoning
- Part III AI GAMES and Searching
- Part IV Model Evaluation and Selection
- Part V Machine Learning
- Part VI Neural Networks

Neural Networks

- *Brief review*
- Feedforward Neural Networks
- Recurrent Neural Networks
- The Learning of Neural Networks
- Model Performance: Cost Function
- Steepest Descent Method
- Backpropagation

Brief review

Artificial Neuron

Biological neural network

- Neurons
- Synaptic Connection
- Biological Neural Networks
- Learning

Abstract

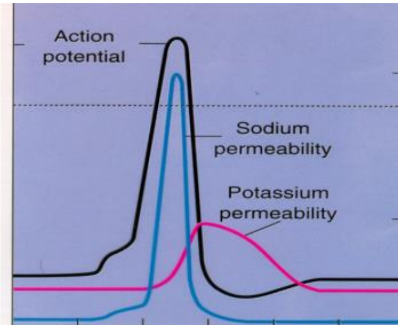
Build a computable mathematical model

Artificial neural networks

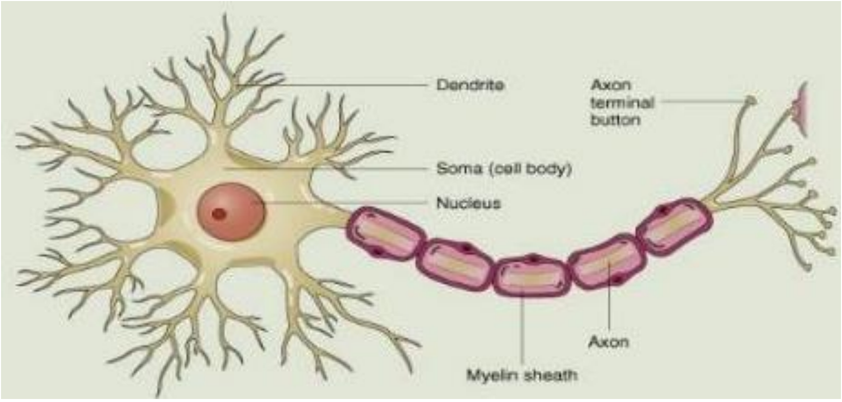
- Neuron model
- Weights
- Neural network models
- Learning algorithm

Computational Model of Neural Network

Artificial Neuron



Single neuron structure

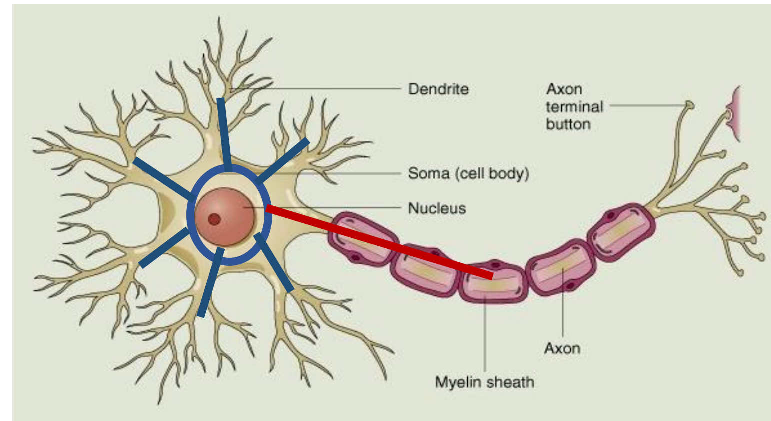


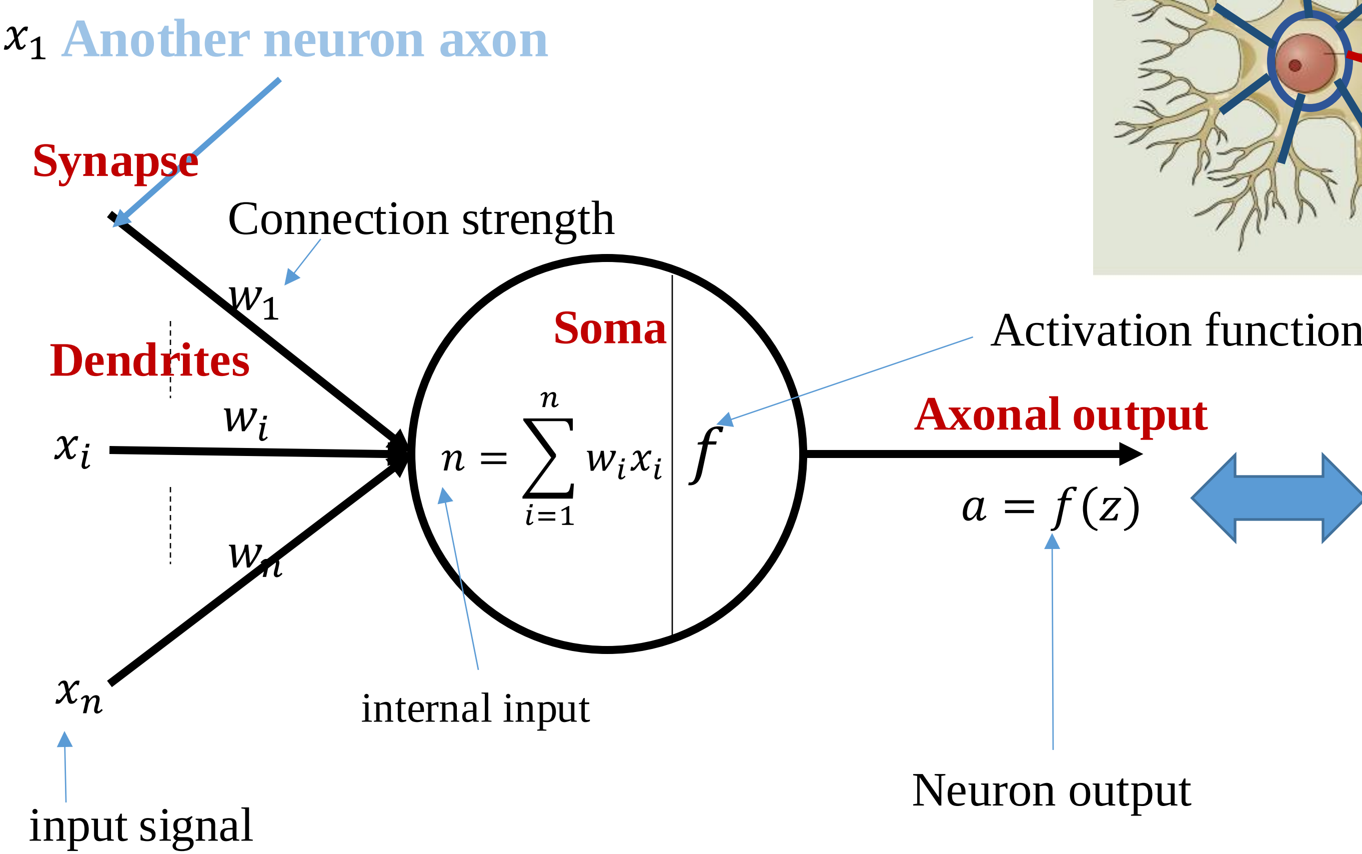
- Soma, Dendrites, Axons
- Function: Collect and transmit signals
- Dendrites receive multiple inputs
- Soma superimposes input information
- Pulses are generated when information is superimposed to a certain extent
- Single output

How to abstract?

Brief review

Artificial Neuron





$$a = f\left(\sum_{i=1}^{R} w_i x_i\right)$$

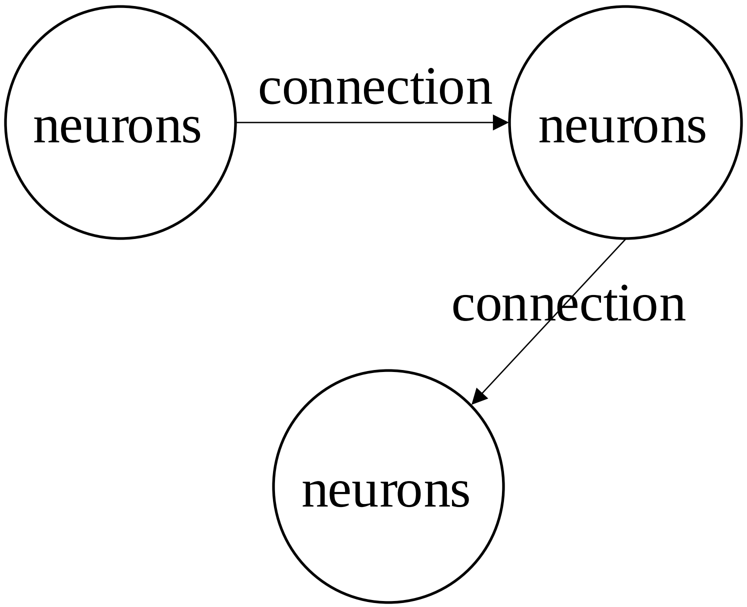
Computational Model of Neural Network

Neural Networks

Feedforward neural network

=

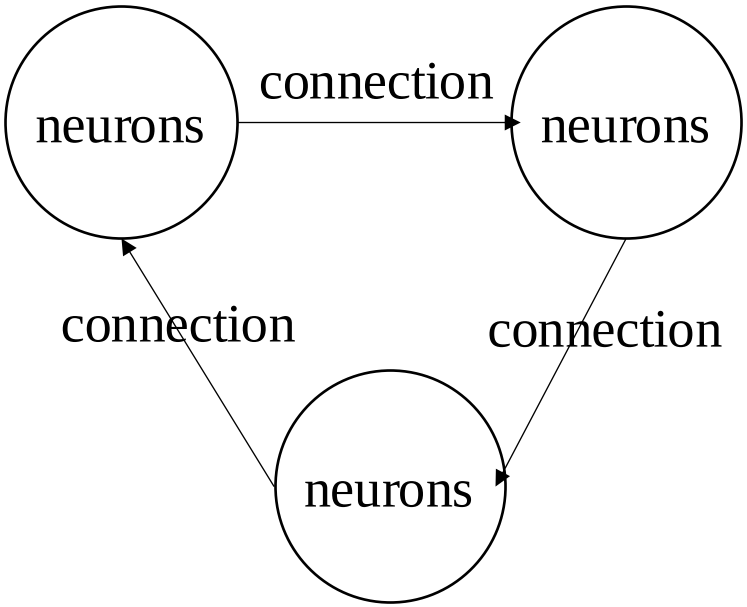
neurons + feedforward connections



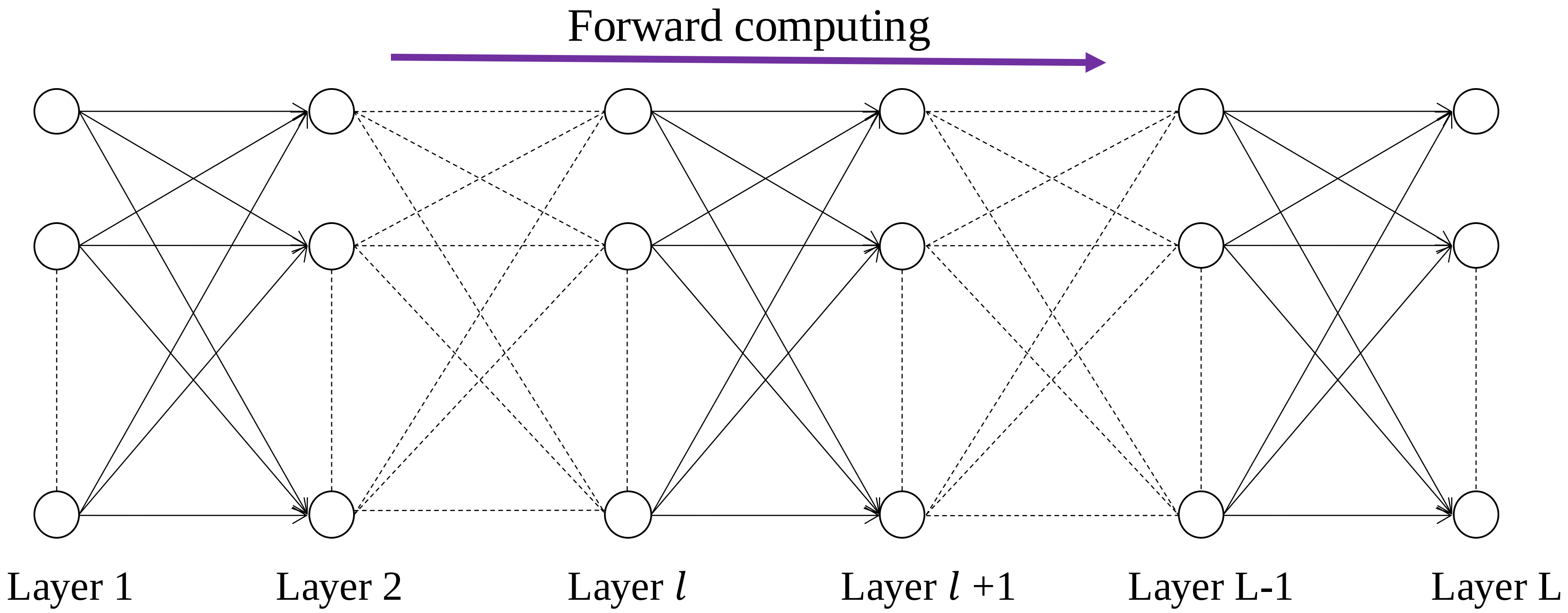
Recurrent neural network

=

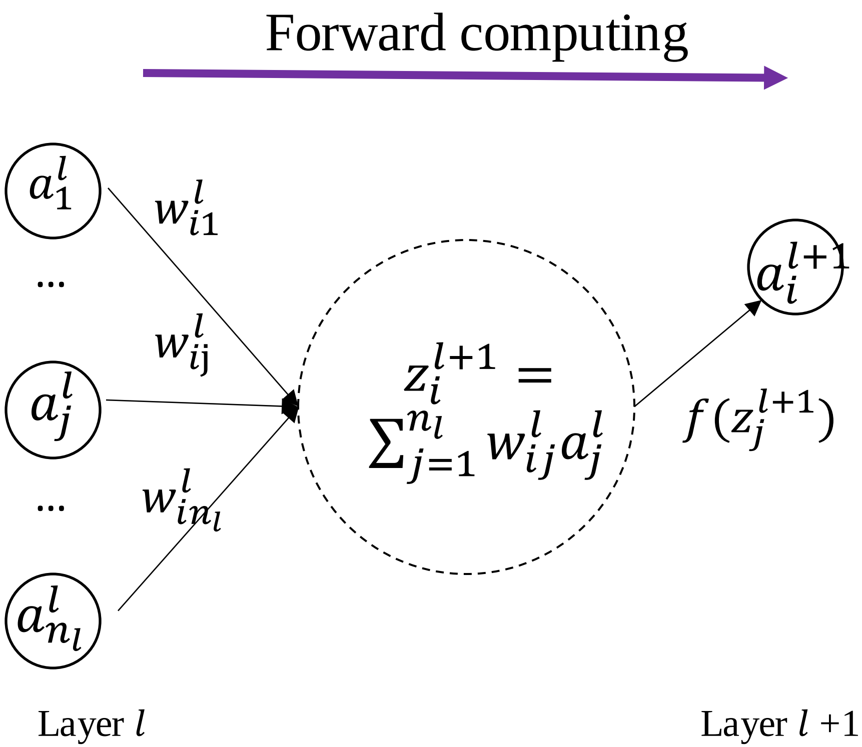
neurons + recurrent connections

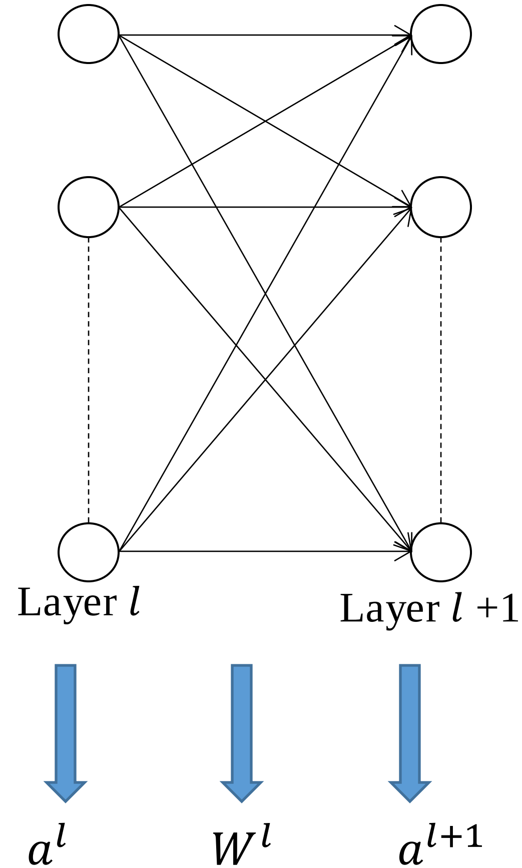


Feedforward Neural Network



Feedforward Neural Network

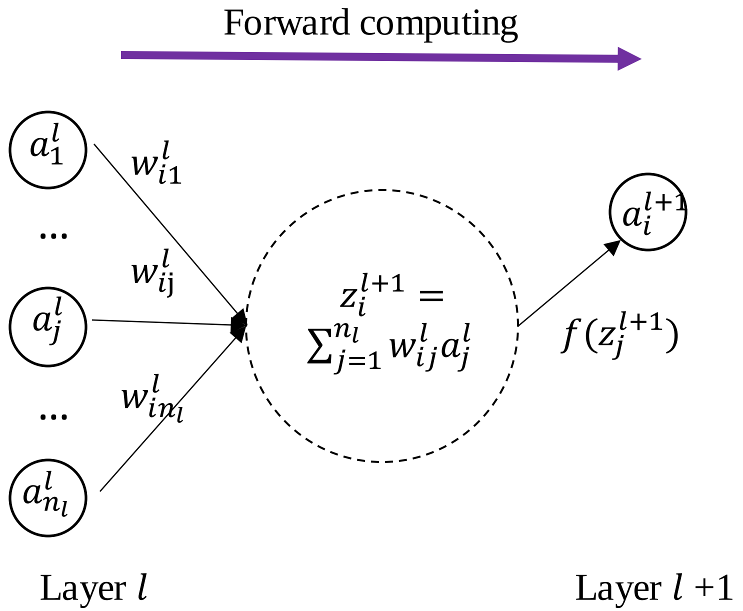




Component form
$$\begin{cases} a_i^{l+1} = f(z_i^{l+1}) \\ z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l \end{cases}$$

Vector form
$$\begin{cases} a^{l+1} = f(z^{l+1}) \\ z^{l+1} = W^l a^l \end{cases}$$

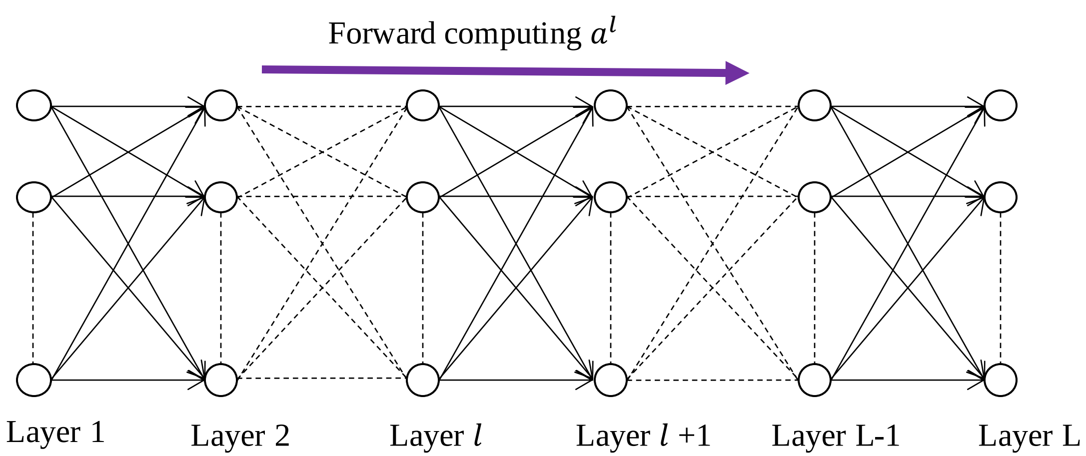
Feedforward Neural Network



Algorithm:

Input W^l, a^l
for l = 1:L, run function:
$\quad a^{l+1} = fc(W^l, a^l)$
return

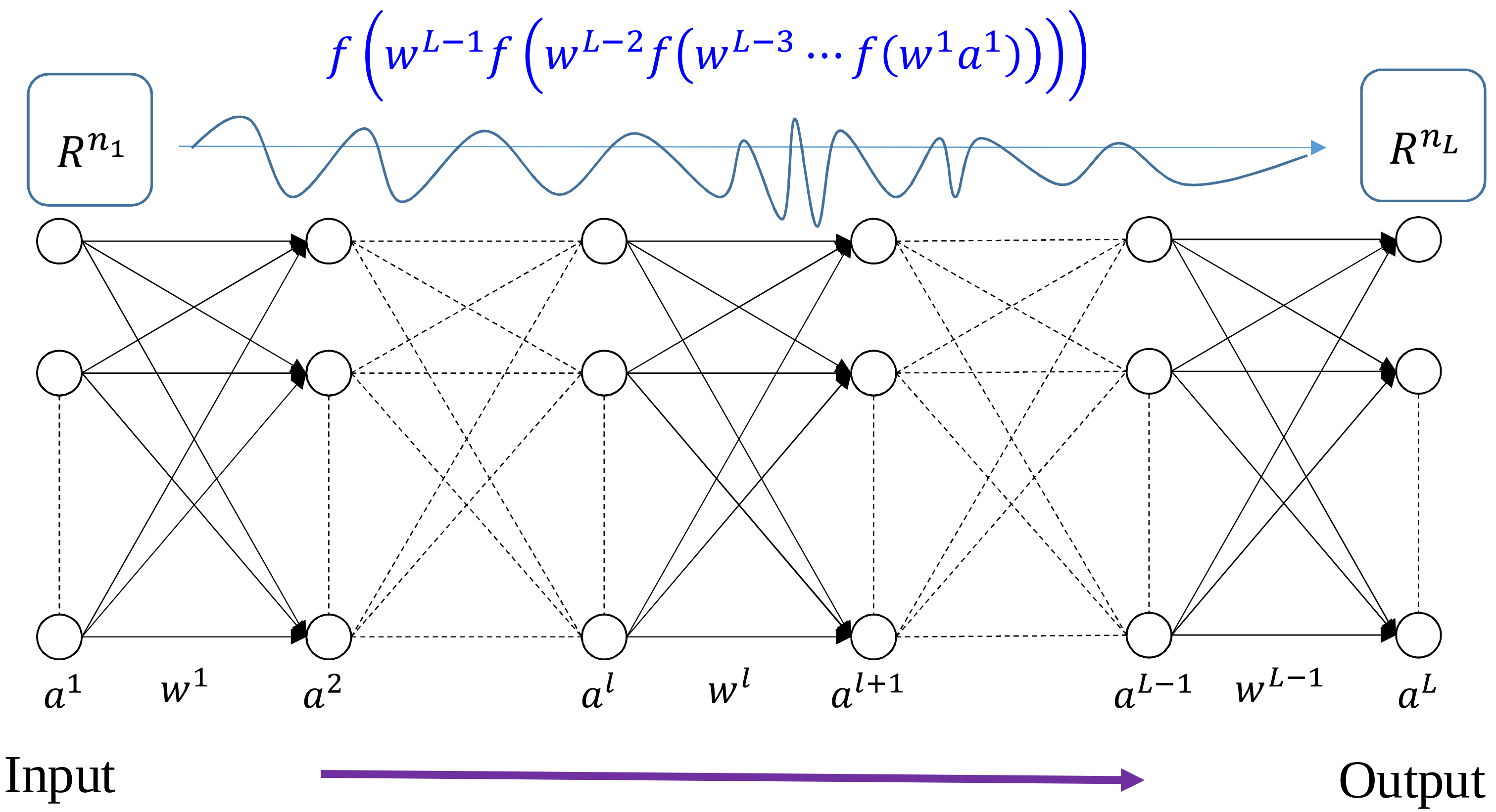
Function $fc(W^l a^l)$
For i = 1: n_{l+1}
$\quad z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$
$\quad a_i^{l+1} = f(z_i^{l+1})$
end



Feedforward Neural Network

In fact, FNN is a nonlinear mapping from R^{n_1} space to R^{n_L} space.

$$a^L = f(w^{L-1}a^{L-1}) = f\left(w^{L-1}f\left(w^{L-2}f\left(w^{L-3}\cdots f(w^1a^1)\right)\right)\right)$$

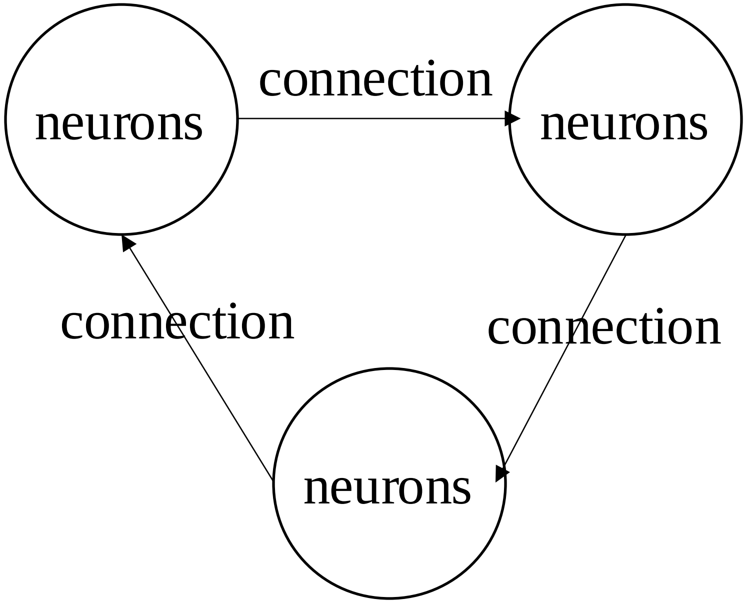


Recurrent Neural Networks

Recurrent neural network

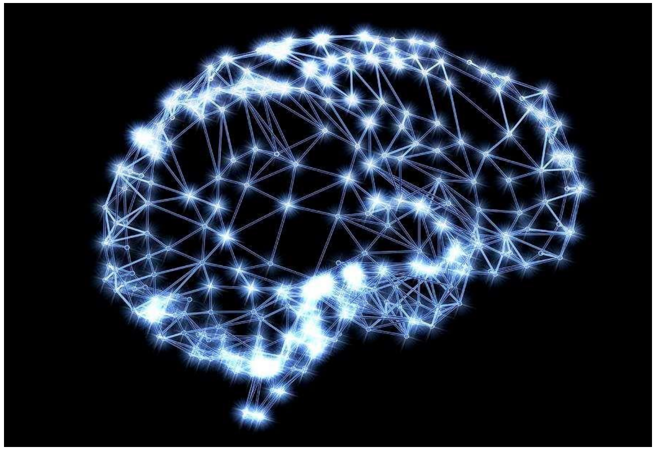
=

neurons + recurrent connections

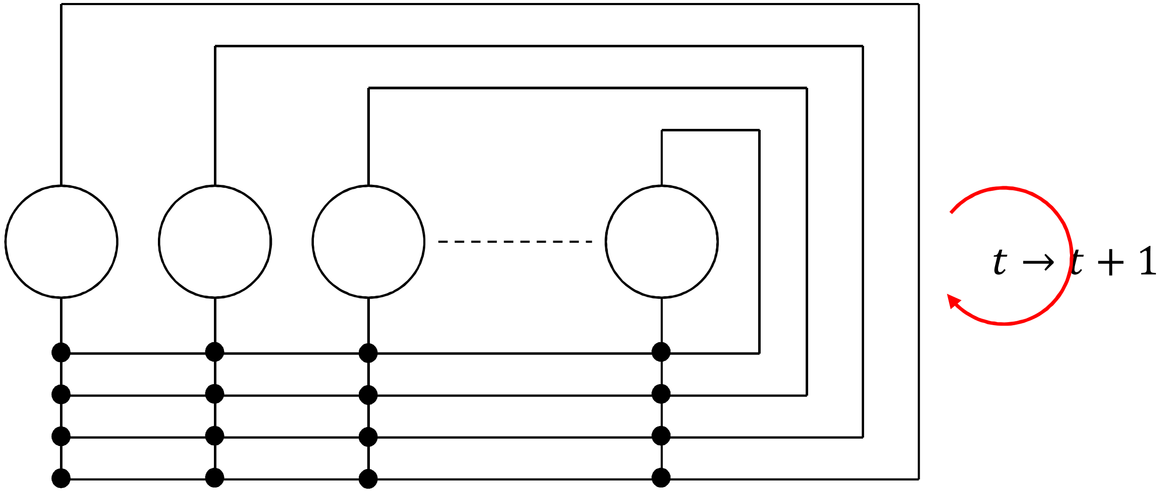


RNNs ---- with feedback connections

Recurrent Neural Networks

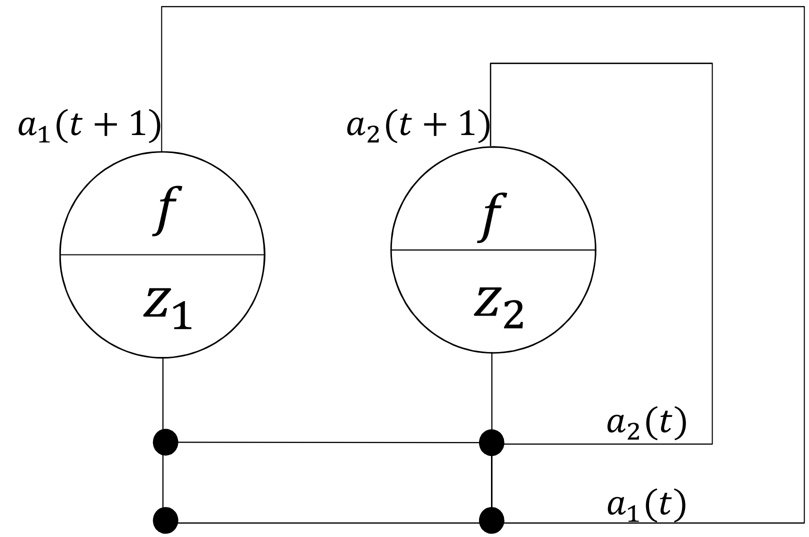


Topology Structure



Problem: how to develop computational model of the RNNs ?

Recurrent Neural Networks



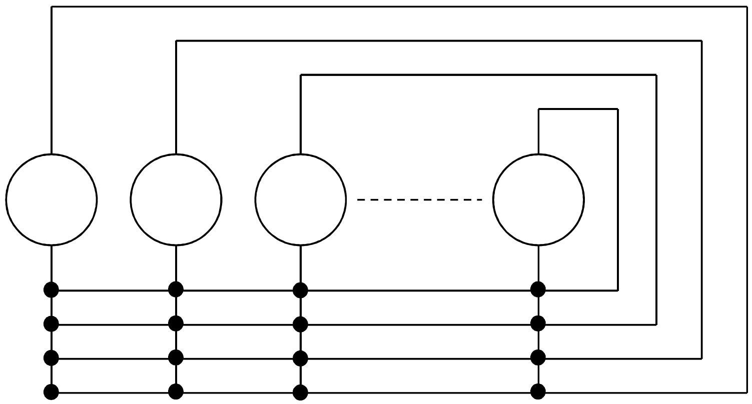
RNNs – Computational Neural Networks Model:

$$\begin{cases} a_1(t+1) = f\big(w_{11}a_1(t) + w_{12}a_2(t)\big) \\ a_2(t+1) = f(w_{21}a_1(t) + w_{22}a_2(t)) \end{cases}$$

Recurrent Neural Networks

Computational Model of RNNs:

$$a_i(t+1) = f\left(\sum_{j=1}^{n} w_{ij} a_j(t)\right)$$



Vector form:

$$a(t+1) = f(Wa(t))$$

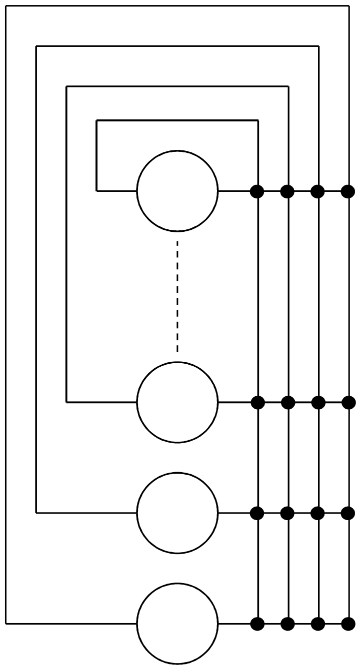
$$W = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix}, a(t) = \begin{bmatrix} a_1(t) \\ \vdots \\ a_n(t) \end{bmatrix}$$

$t \to t+1$

The time changes in discrete manner.

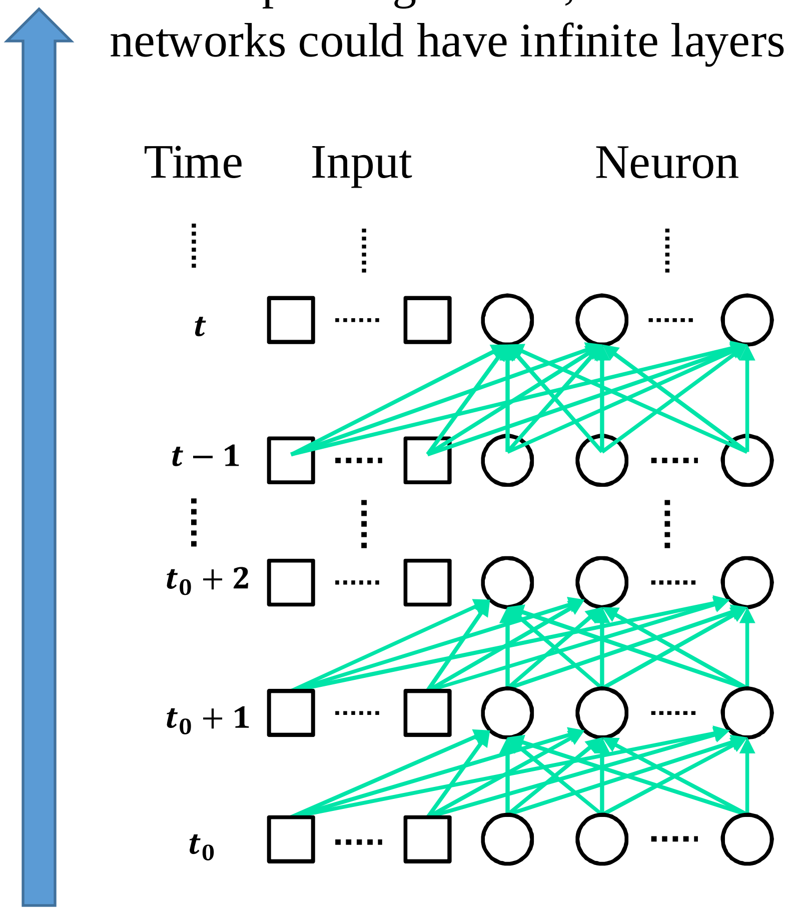
This model is a discrete time dynamic system.

Recurrent Neural Networks

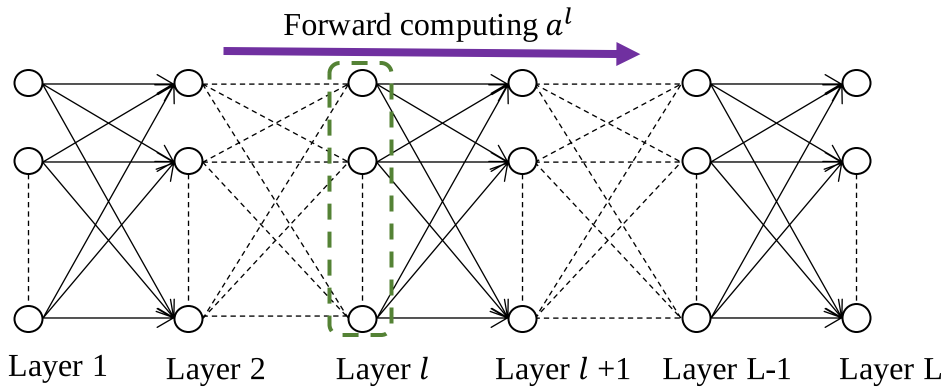


RNN could be expanded in time dimension.

With expanding in time, this networks could have infinite layers.



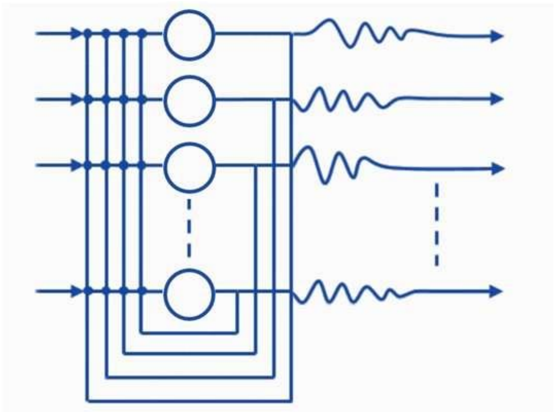
FNNs VS. RNNs



no recurrent connection

FNNs

- Extract the spatial features of static data
- Describe spatial correlation



with recurrent connection

RNNs

- Memory mechanism
- Extract spatiotemporal features of time sequence data
- Describe time correlation

Neural Networks

- Brief review
- Feedforward Neural Networks
- Recurrent Neural Networks
- *The Learning of Neural Networks*
- Model Performance: Cost Function
- Steepest Descent Method
- Backpropagation

The Learning of Neural Networks

- Knowledge is acquired by learning.
  - Three human learning models:

Learning with teacher

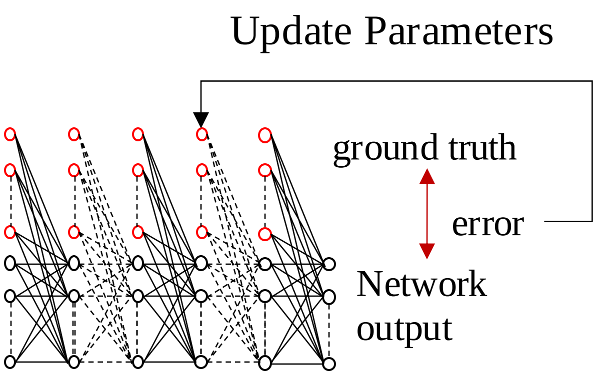
Reinforcement learning

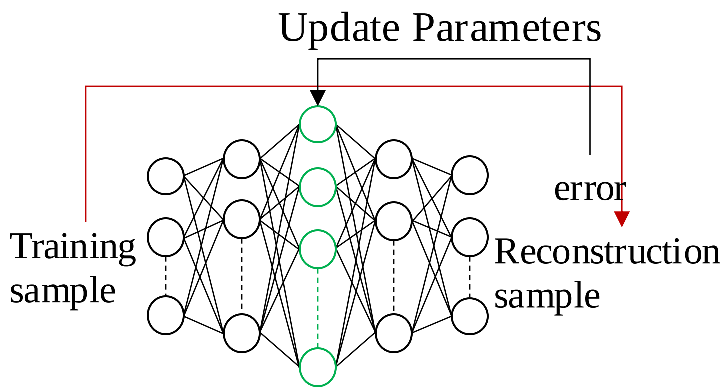
Learning without teacher

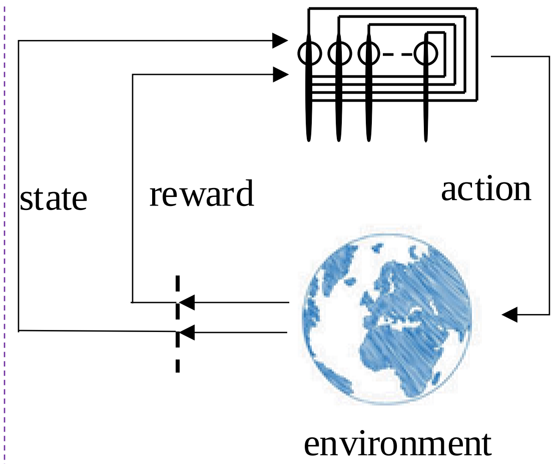
Learning: establishment of new connections and the modification of existing connections

The Learning of Neural Networks

- Learning is to change the connections by some rules.
- Similar with the three learning model of human:







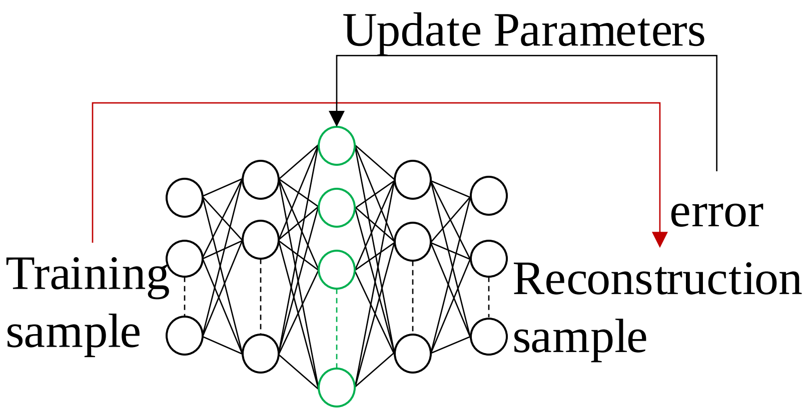
Supervised Learning: Update the network parameters according to the error between the target output and the actual network output of the training sample

Unsupervised learning: For non-label samples, the network parameters are updated by reconstructing these samples.

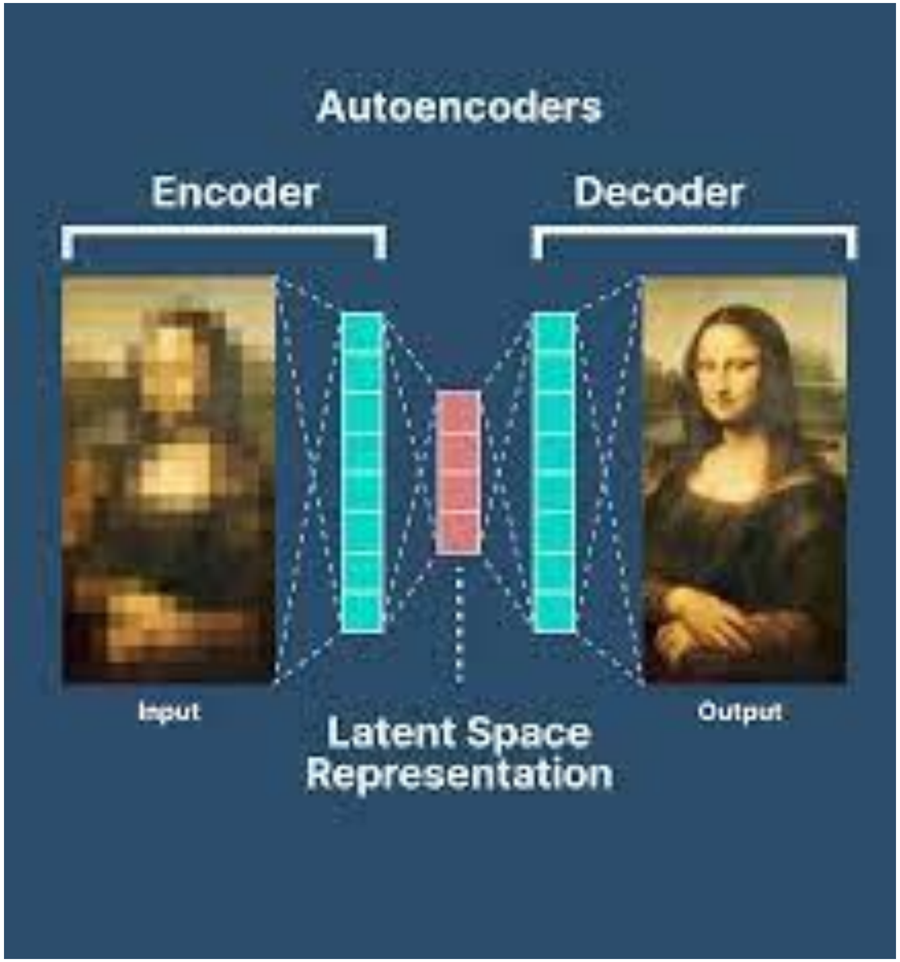
Reinforcement learning: Update network parameters with the goal of maximizing rewards during interactions with the environment

The Learning of Neural Networks

Unsupervised Learning



Unsupervised learning: For non-label samples, the network parameters are updated by reconstructing these samples.

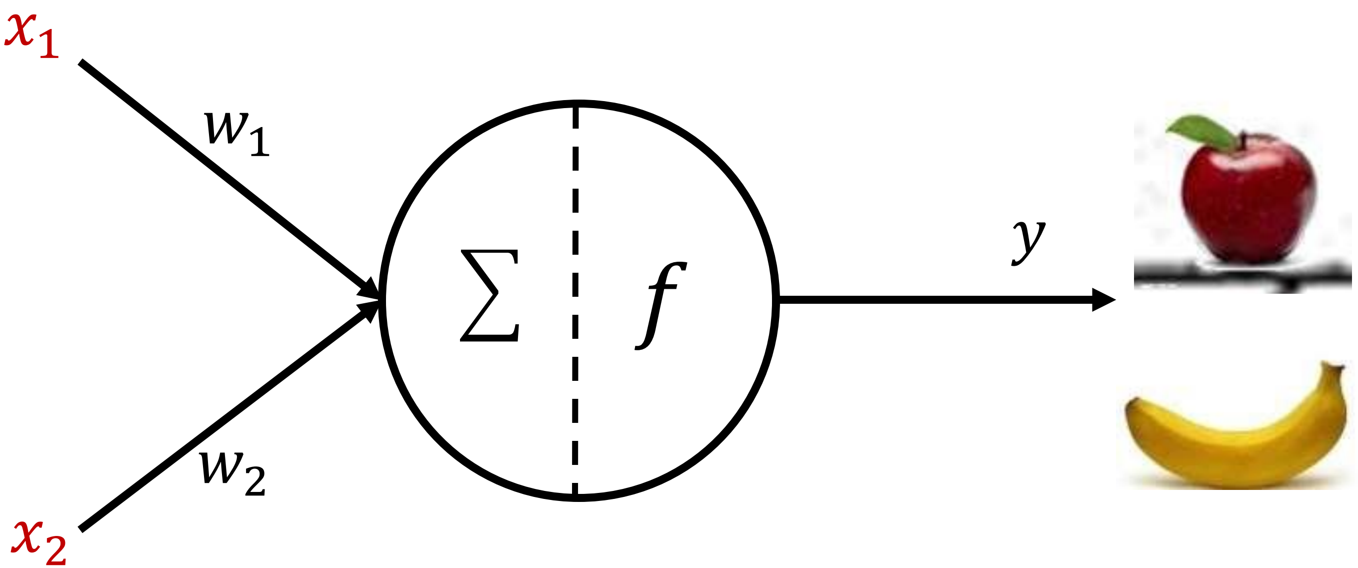


The Learning of Neural Networks

- Supervised Learning

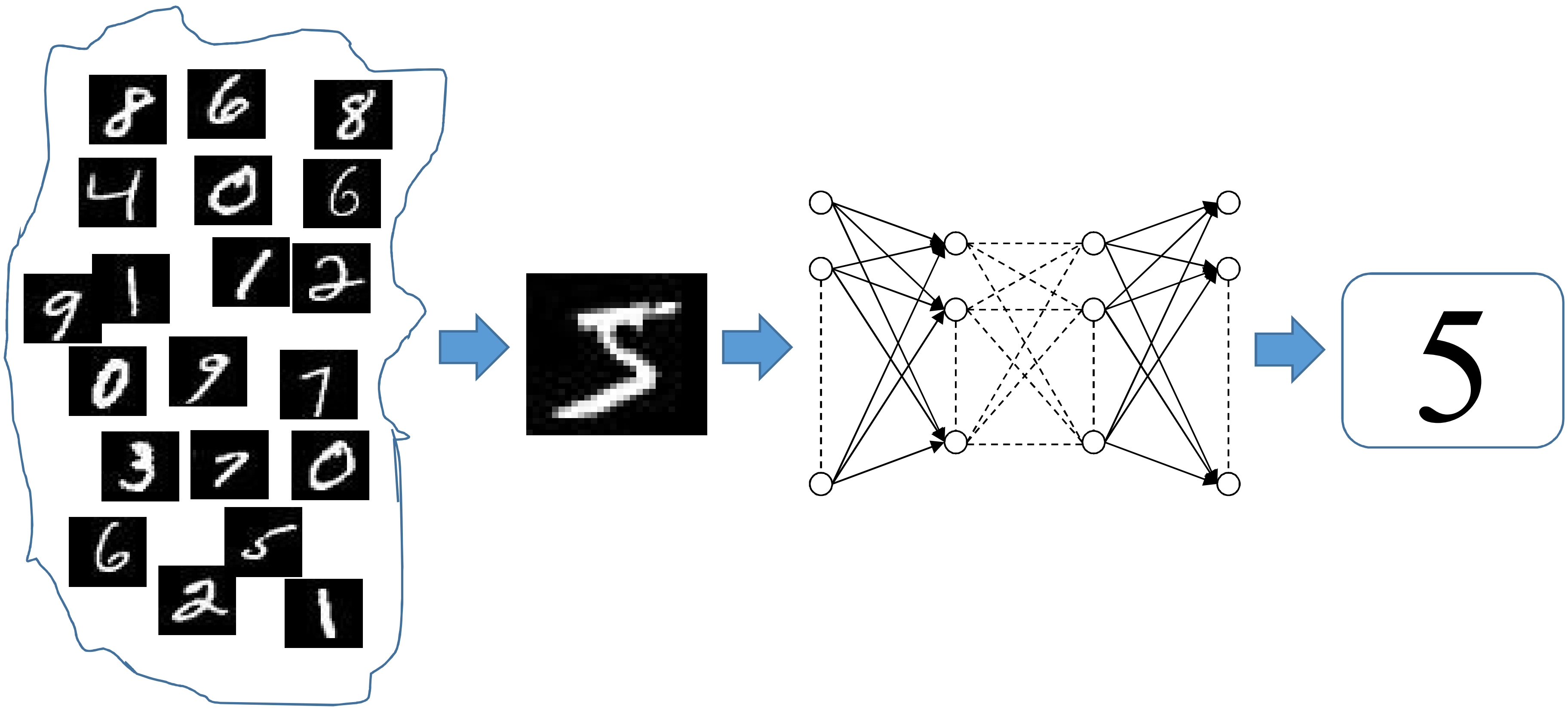
Feature: red, round

Feature: yellow, strip



The Learning of Neural Networks

- Supervised Learning

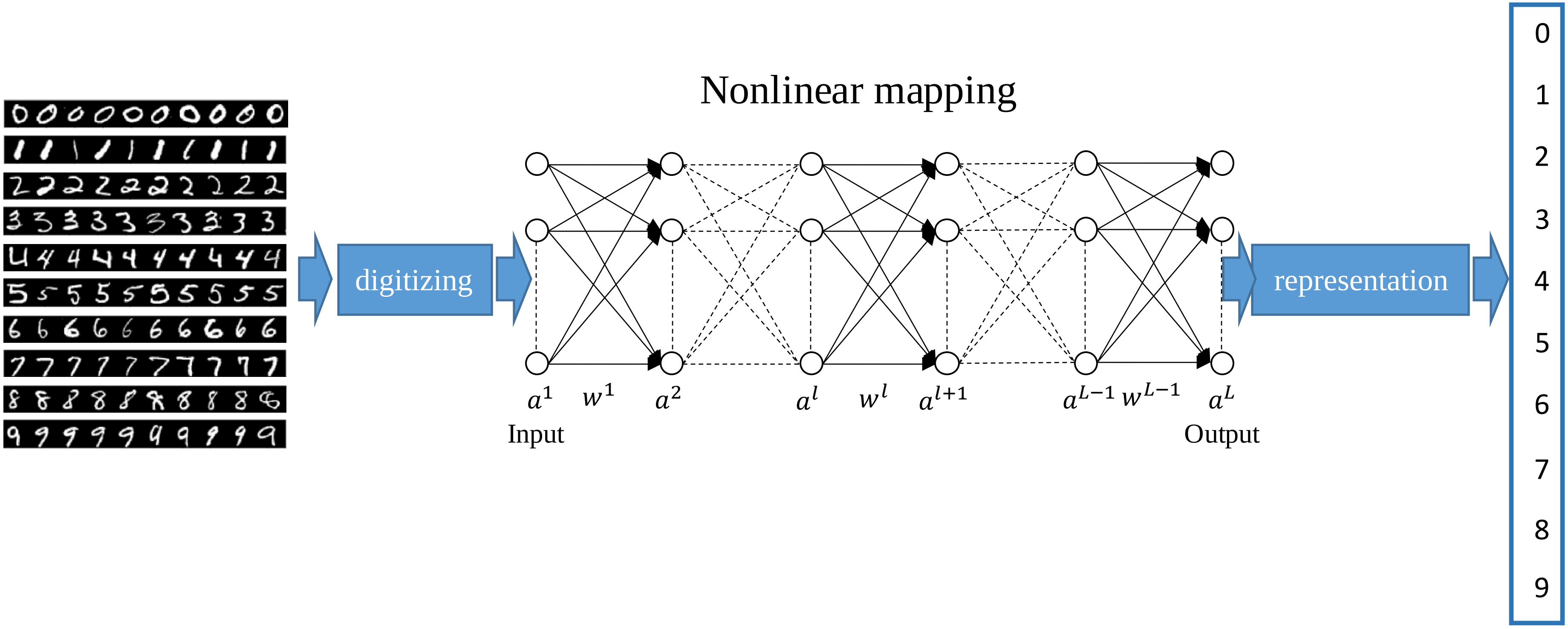


Neural Networks

- Brief review
- Feedforward Neural Networks
- Recurrent Neural Networks
- The Learning of Neural Networks
- *Model Performance: Cost Function*
- Steepest Descent Method
- Backpropagation

Model Performance: Cost Function

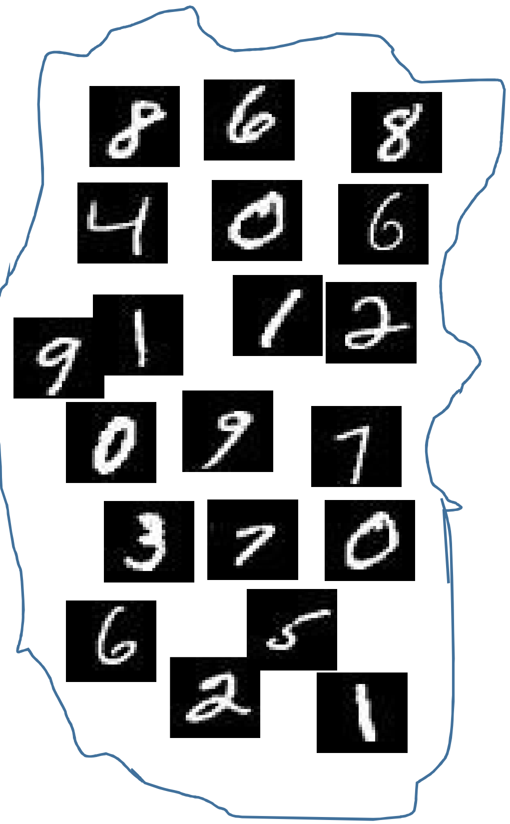
Nonlinear Mapping

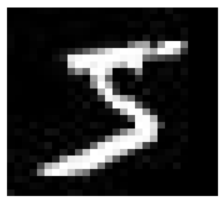


Problem: How to design the NN? Are there any methods to find "good" connection weights?

Model Performance: Cost Function

Cost Function





Good performance!
The mother knows correct answer.

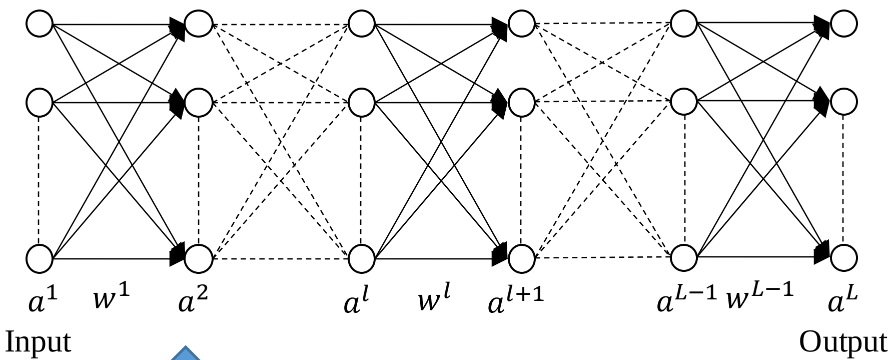
5

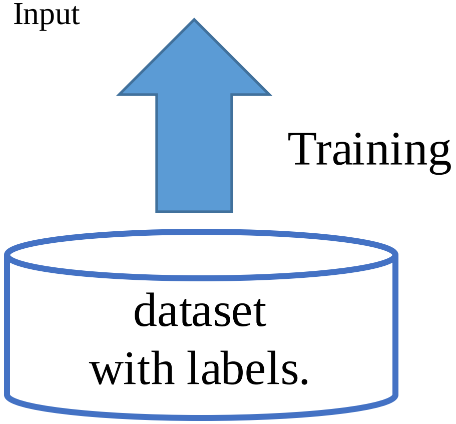
Two important factors:

1. There must be a measure to measure the correctness between correct answer and the girl’s real output. ----- Performance function (性能函数) .
2. There must be a mechanism to change the knowledge system of the girl. ---- Learning algorithm (学习算法) .

Model Performance: Cost Function

Cost Function





The goal of Learning:
Network output ≈ Target output

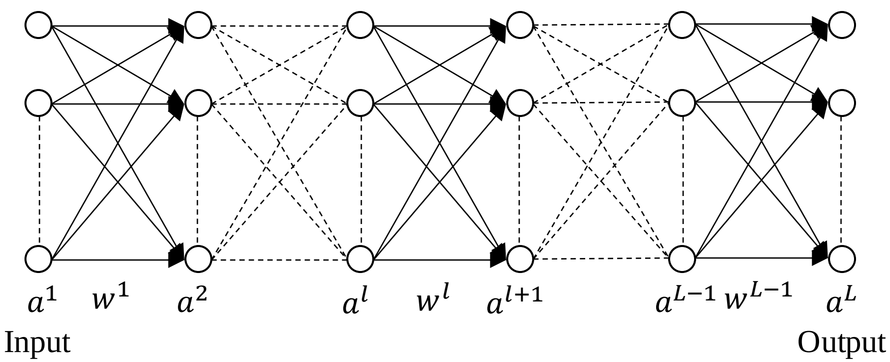
Cost Function $J(a^L, y^L)$:

- describe the distance between network output a^L and target output y^L
- $J(a^L, y^L)$ is a function related to $(w^1, \cdots, w^{L-1})$

$$J = J(w^1, \cdots, w^{L-1})$$

Model Performance: Cost Function

Cost Function



Target Output	Network Output
$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$	$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$

- The cost function describes the performance of the network. The smaller J is, the closer the network output is to the target output, and the better the network performance is.
- $J(a^L, y^L)$ is a function related to $(w^1, \cdots, w^{L-1})$, to get a good performance is to find a good $(w^1, \cdots, w^{L-1})$.
- To find the good $(w^1, \cdots, w^{L-1})$ is the learning of neural network.

Model Performance: Cost Function

Cost Function

Learning is a process such that a^L is close to y^{L}, i.e., the cost function J reaches minimum.

Target Output	Network Output
$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$	$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$

A cost function $J = J(w^1, \cdots, w^{L-1})$ is a function with variables $w^l (l = 1, \cdots, L-1)$, thus the network learning is to looking for some $w^l (l = 1, \cdots, L-1)$ such that $w^l (l = 1, \cdots, L-1)$ is a minimum point of J.

A frequently used cost function:

$$J = \frac{1}{2} \sum_{j=1}^{n_L} e_j^2 = J(w^1, \cdots, w^{L-1})$$

J is a function of $w^1, \cdots, w^{L-1}$.

Problem: How to find out the minimum points of J ?

Learning = Looking for minimum points of J

Neural Networks

- Brief review
- Feedforward Neural Networks
- Recurrent Neural Networks
- The Learning of Neural Networks
- Model Performance: Cost Function
- *Steepest Descent Method*
- Backpropagation

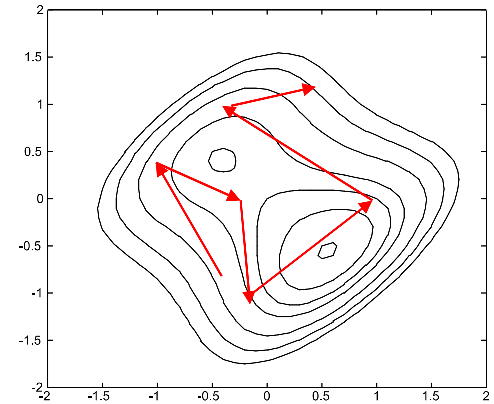
Steepest Descent Method

Minimum Points

General Nonlinear function

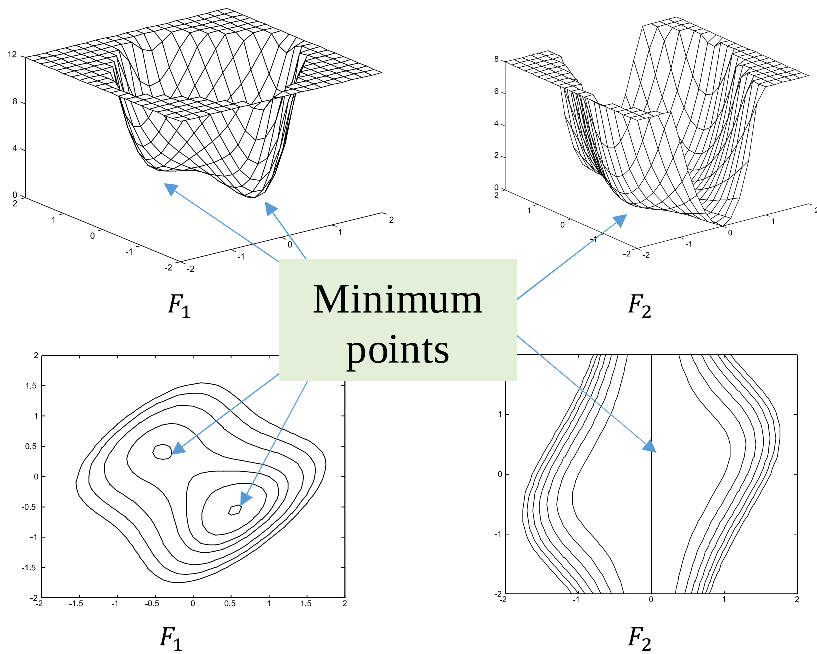
$$F(x), x \in R^n$$

x^* is a minimum point if $F(x^*) \leq F(x)$ for any x that very close to x^*.



$$F_1(w) = (w_2 - w_1)^4 + 8w_1w_2 - w_1 + w_2 + 3$$

$$F_2(w) = (w_1^2 - 1.5w_1w_2 + 2w_2^2)w_1^2$$



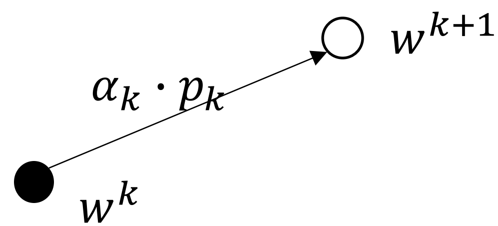
Iteration Methods:

1. Setting a starting point x_0
2. Finding a minimum point step by step:

$$w^{k+1} = w^k + \alpha_k \cdot p_k,$$

p_k: is called searching direction

α_k: is leaning rate at step k

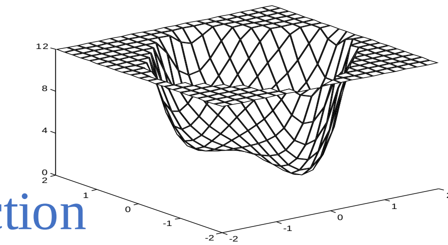


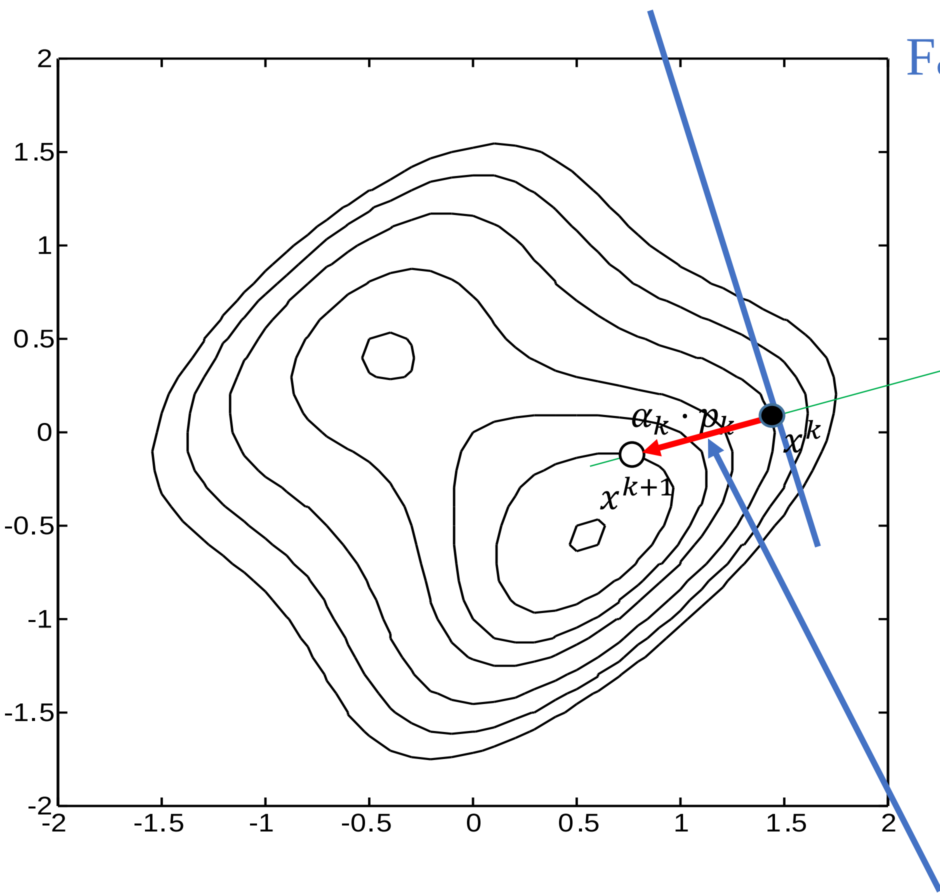
Problem: how to get the searching direction p_k.

Steepest Descent Method

Slowest changing direction

Fastest increasing direction





Gradient:

$$g_k = \nabla F(w)\Big|_{w^k} = \frac{\partial F}{\partial w}\Big|_{w^k} = \begin{pmatrix} \frac{\partial F}{\partial w_1} \\ \vdots \\ \frac{\partial F}{\partial w_n} \end{pmatrix}\Bigg|_{w^k}$$

Steepest Descent Algorithm:

$$p_k = -g_k$$
$$w^{k+1} = w^k - \alpha_k \cdot g_k$$

or

$$w^{k+1} = w^k - \alpha_k \cdot \frac{\partial F}{\partial w}\Big|_{w^k}$$

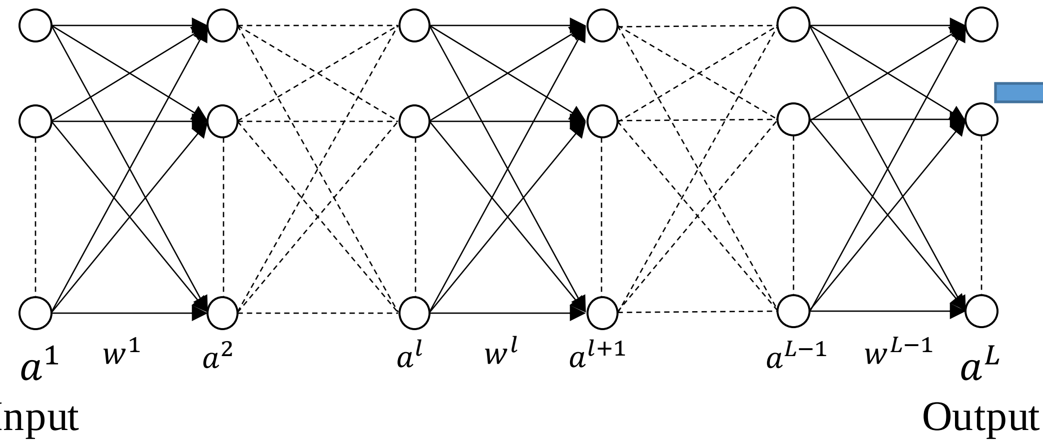
Steepest descent direction

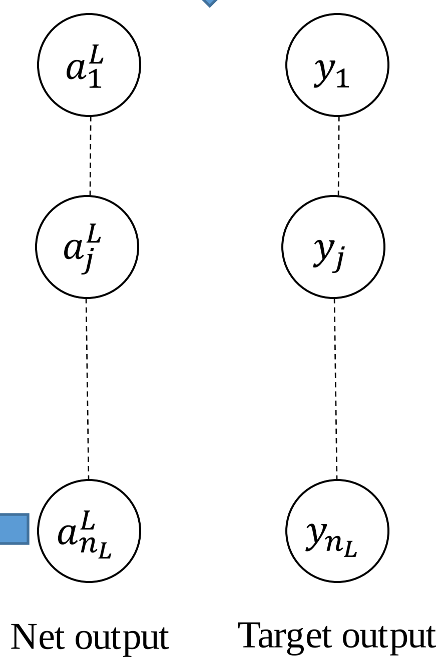
Steepest Descent Method

- Deep learning

Steepest Descent Algorithm:

$$w^{k+1} = w^k - \alpha_k \cdot \left.\frac{\partial F}{\partial w}\right|_{w^k}$$





Updating weights

$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$$

Computing gradient

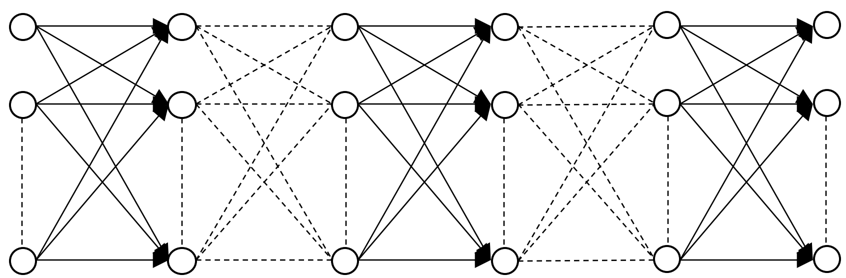
$$\frac{\partial J}{\partial w_{ji}^l}$$

Construct cost function

$$J = \frac{1}{2}\sum_{j=1}^{n_L}\left(y_j - a_j^L\right)^2$$

Steepest Descent Method

Deep learning



Target Output	Network Output
$y^L = \begin{bmatrix} y_1^L \\ \vdots \\ y_{n_L}^L \end{bmatrix}$	$a^L = \begin{bmatrix} a_1^L \\ \vdots \\ a_{n_L}^L \end{bmatrix}$

$$a^L = f(W^{L-1}a^{L-1}) = f\left(W^{L-1}f\left(W^{L-2}f\left(W^{L-3}\cdots f(W^1a^1)\right)\right)\right)$$

Steepest Descent Method

$$J = \frac{1}{2}\sum_{j=1}^{n_L} e_j^2 = \frac{1}{2}\sum_{j=1}^{n_L}\left(a_j^L - y_j^L\right)^2$$

1. Computing

$$\frac{\partial J}{\partial w_{ji}^l}$$

2. Iterating

$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$$

Problem: How to compute $\frac{\partial J}{\partial w_{ji}^l}$?

Answer:
Using the well-known BP method.

Neural Networks

- Brief review
- Feedforward Neural Networks
- Recurrent Neural Networks
- The Learning of Neural Networks
- Model Performance: Cost Function
- Steepest Descent Method
- *Backpropagation*

Backpropagation

- Updating weights: compute $\dfrac{\partial J}{\partial w_{ji}^{l}}$

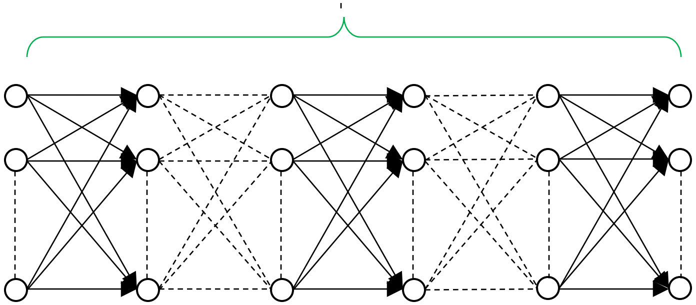
l layer

$a_i^l = f(z_i^l)$

define $\delta_i^l = \dfrac{\partial J}{\partial z_i^l}$

Problem: What's the relation between δ_i^l and $\dfrac{\partial J}{\partial w_{ji}^{l}}$?

$J(W^1, \cdots, W^{L-1})$



l layer

$a_i^l = f(z_i^l)$

$\delta_i^l = \dfrac{\partial J}{\partial z_i^l}$

w_{ji}^l

$z_j^{l+1} = \sum_{i=1}^{n_l} w_{ji}^l a_i^l$

$l+1$ layer

$a_j^{l+1} = f(z_j^{l+1})$

$\delta_j^{l+1} = \dfrac{\partial J}{\partial z_j^{l+1}}$

Relation between δ_i^l and $\dfrac{\partial J}{\partial w_{ji}^{l}}$

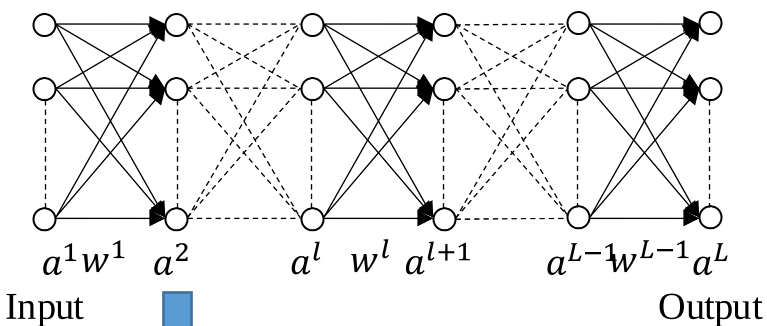
$$\frac{\partial J}{\partial w_{ji}^{l}} = \delta_j^{l+1} \cdot a_i^l$$

Why?

$$\frac{\partial J}{\partial w_{ji}^{l}} = \frac{\partial J}{\partial z_j^{l+1}} \cdot \frac{\partial z_j^{l+1}}{\partial w_{ji}^{l}} = \delta_j^{l+1} \cdot a_i^l$$

Backpropagation

Updating weights



Construct cost function

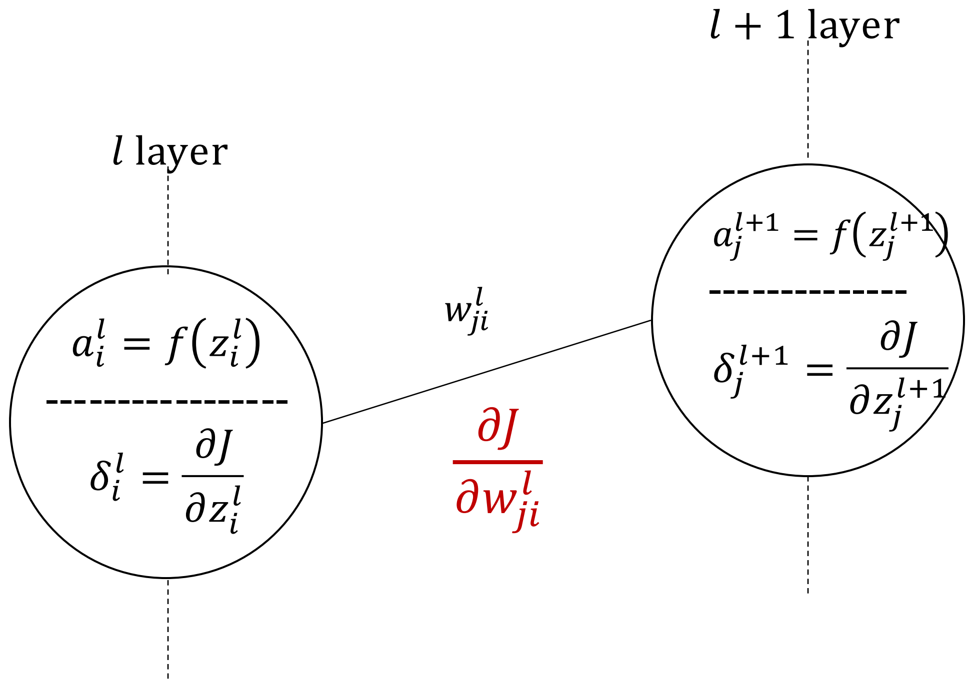
$$J = \frac{1}{2}\sum_{j=1}^{n_L}(y_j - a_j^L)^2$$

Updating weights

$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$$

$$\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$$

$$\delta_i^{l+1} = \frac{\partial J}{\partial z_i^{l+1}}$$



Problem:

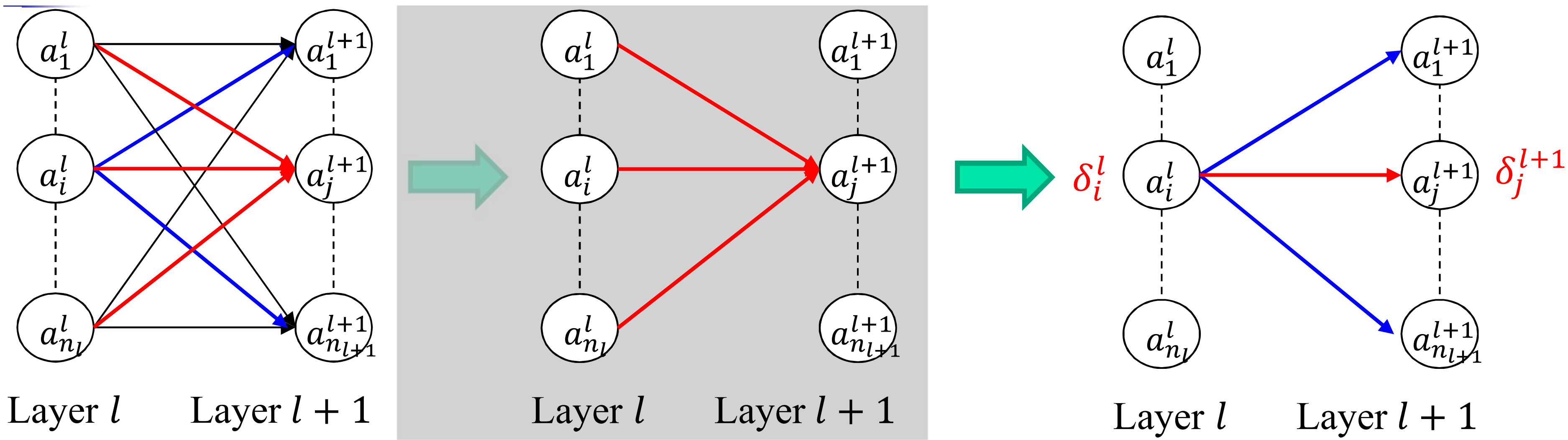
How to compute δ_i^l?

Because, it's easy to compute $\delta_i^L = \frac{\partial J}{\partial z_i^L}$,

What's the relation between δ_i^l and δ_j^{l+1} ?

Backpropagation

Updating weights



The relation between δ_i^l and δ_j^{l+1}

$$\boxed{z_j^{l+1} = \sum_{i=1}^{n_l} w_{ji}^l a_i^l = \sum_{i=1}^{n_l} w_{ji}^l f\left(z_i^l\right)}$$

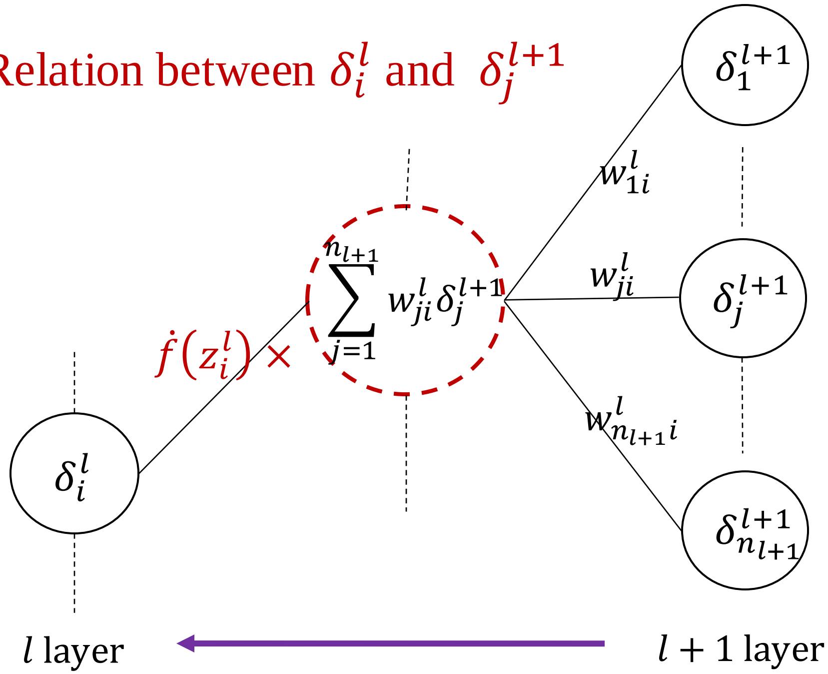
$$\delta_i^l = \frac{\partial J}{\partial z_i^l} = \sum_{j=1}^{n_{l+1}} \frac{\partial J}{\partial z_j^{l+1}} \cdot \frac{\partial z_j^{l+1}}{\partial z_i^l} = \sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \boxed{\frac{\partial z_j^{l+1}}{\partial z_i^l}} = \sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot w_{ji}^l \, \dot{f}\left(z_i^l\right) = \dot{f}\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot w_{ji}^l\right)$$

Backpropagation

Updating weights

$$\delta_i^l = \frac{\partial J}{\partial z_i^l} = \sum_{j=1}^{n_{l+1}} \frac{\partial J}{\partial z_j^{l+1}} \cdot \frac{\partial z_j^{l+1}}{\partial z_i^l} = \sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot \frac{\partial z_j^{l+1}}{\partial z_i^l} = \sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot w_{ji}^l \dot{f}(z_i^l) = \dot{f}(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot w_{ji}^l \right)$$

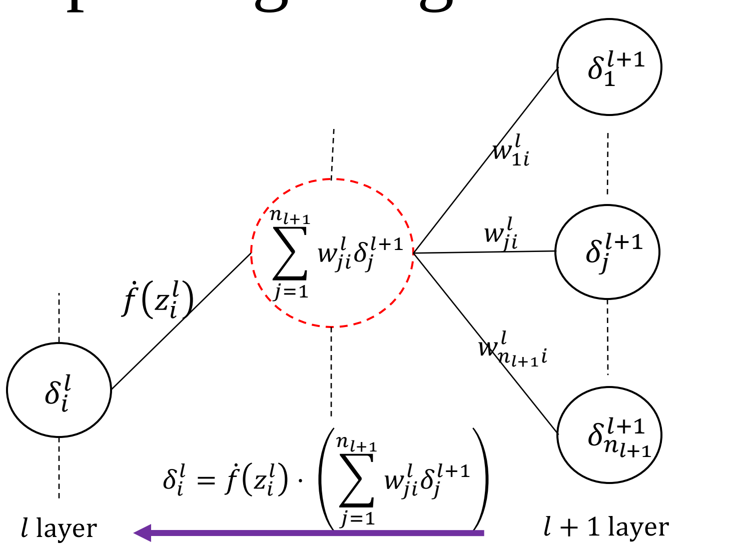
Relation between δ_i^l and δ_j^{l+1}

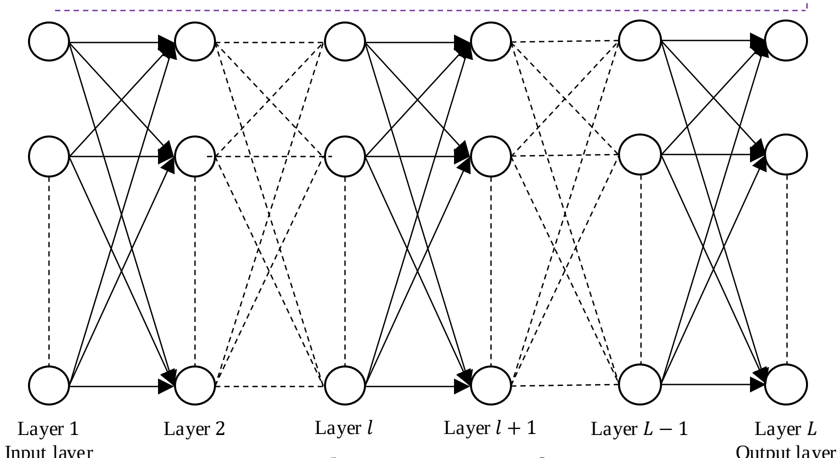


$$\delta_i^l = \dot{f}(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1} \right)$$

Backpropagation

Updating weights





Backpropagation δ

Relation between δ_i^l and δ_j^{l+1}

$$\delta_i^l = \dot{f}(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot w_{ji}^l\right)$$

$$\delta_i^L = \frac{\partial J}{\partial z_i^L}$$

If

$$J = \frac{1}{2}\sum_{j=1}^{n_L}\left(a_j^L - y_j^L\right)^2$$

then,

$$\delta_i^L = \frac{\partial J}{\partial z_i^L} = \left(a_i^L - y_i^L\right) \cdot \frac{\partial a_j^L}{\partial z_i^L}$$
$$= \left(a_i^L - y_i^L\right) \cdot \dot{f}(z_i^L)$$

Backpropagation

Conclusion: BP for FNN

Forward computing: $y = f\left(\sum_{i=1}^{n} w_i x_i\right)$

Define cost function: $J = J(w^1, \cdots, w^{L-1})$

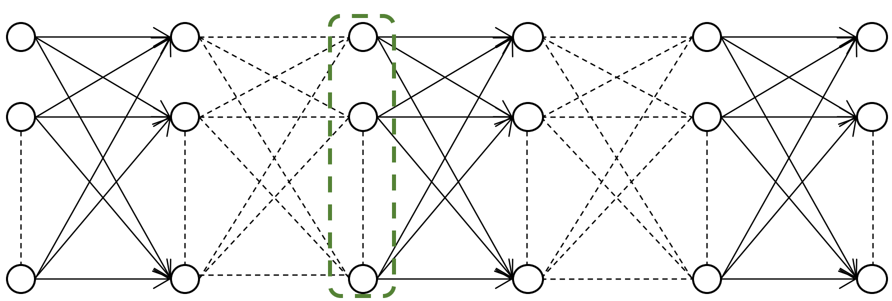
Updating rule: $w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$

Define δ: $\delta_i^l = \frac{\partial J}{\partial z_i^l}$

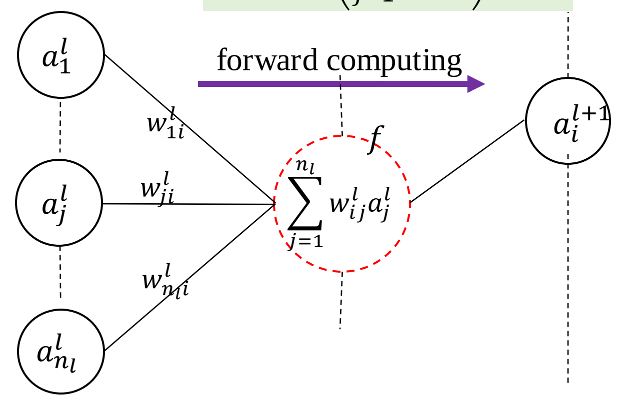
Find the relation: $\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$

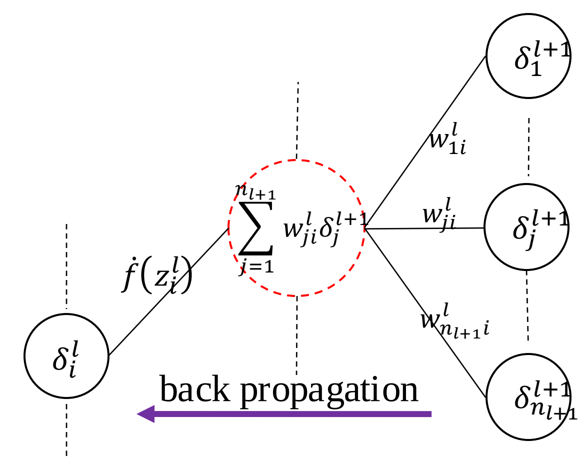
Back propagation: $\delta_i^L = \frac{\partial J}{\partial z_i^L} = \left(a_i^L - y_i^L\right) \cdot \dot{f}\left(z_i^L\right)$

$$\delta_i^l = \dot{f}\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} \delta_j^{l+1} \cdot w_{ji}^l\right)$$



$$a_i^{l+1} = f\left(\sum_{j=1}^{n_l} w_{ij}^l a_j^l\right)$$



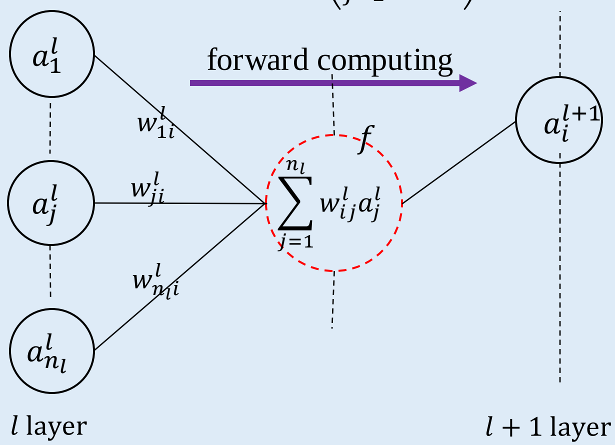


$$\delta_i^l = \dot{f}\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$

Backpropagation

Conclusion: BP for FNN

$$a_i^{l+1} = f\left(\sum_{j=1}^{n_l} w_{ij}^l a_j^l\right)$$



function $fc\left(W^l, a^l\right)$

$for\ i = 1{:}\,n_{l+1}$

$$z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$$

$$a_i^{l+1} = f\left(z_i^{l+1}\right)$$

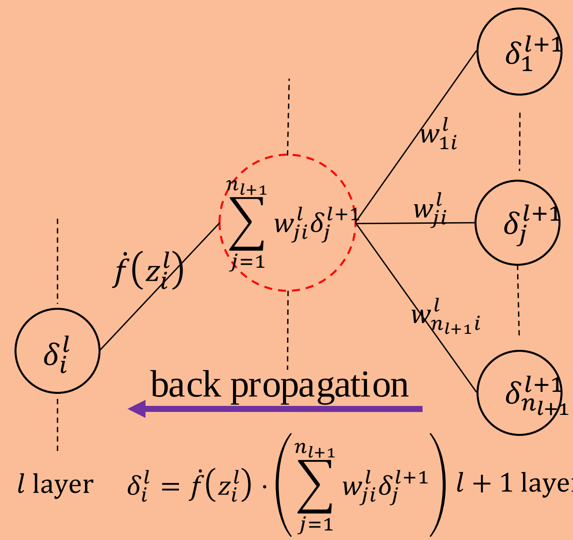
end

function $bc\left(W^l, \delta^{l+1}\right)$

$for\ i = 1{:}\,n_l$

$$\delta_i^l = \dot{f}\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$

end



Backpropagation

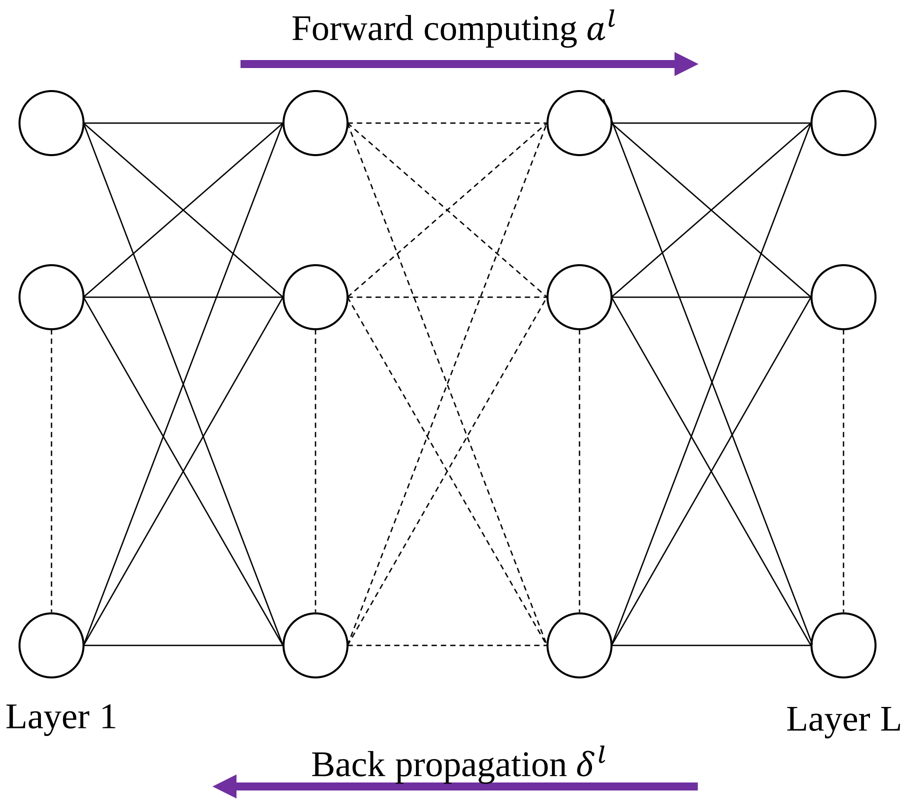
Algorithm

The training data set
$D = \{(x, y) | m\ samples\}$

x: $input\ sample$
y: $target\ output$

There are two ways to train the network.

1. Online training: For each sample $(x, y) \in D$, define a cost function, for example, as
$$J(x, y) = \frac{1}{2}\sum_{j=1}^{n_L}\left(a_j^L - y_j^L\right)^2$$
2. Batch training: Define cost function as
$$J = \frac{1}{m}\sum_{(x,y)\in D} J(x, y)$$



Backpropagation

Algorithm

Batch BP Algorithm:

Step 1. Input the training data set $D = \{(x, y)\}$

Step 2. Initial each w_{ij}^l, and choose a learning rate α.

Step 3. For all m samples $(x, y) \in D$, set $a^1 = x$

for $l = 1: L-1$

$a^{l+1} \leftarrow fc(w^l, a^l)$

end

$$\delta^L \leftarrow \frac{\partial J}{\partial z^L}$$

for $l = L-1: 1$

$\delta^l \leftarrow bc(w^l, \delta^{l+1})$

end

$$\frac{\partial J}{\partial w_{ji}^l} \leftarrow \frac{\partial J}{\partial w_{ji}^l} + \frac{1}{m}\delta_j^{l+1} \cdot a_i^l$$

Step 4. Updating

$$w_{ji}^l \leftarrow w_{ji}^l - \alpha \cdot \frac{\partial J}{\partial w_{ji}^l}$$

Step 5. Return to Step 3 until each w^lconverge.

function $fc(w^l, a^l)$

$for\ i = 1: n_{l+1}$

$$z_i^{l+1} = \sum_{j=1}^{n_l} w_{ij}^l a_j^l$$

$$a_i^{l+1} = f(z_i^{l+1})$$

end

Relationship:

$$\frac{\partial J}{\partial w_{ji}^l} = \delta_j^{l+1} \cdot a_i^l$$

function $bc(w^l, \delta^{l+1})$

$for\ i = 1: n_l$

$$\delta_i^l = \dot{f}(z_i^l) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1}\right)$$

end