The Introduction To Artificial Intelligence

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The Introduction to Artificial Intelligence

- Part I Brief Introduction to AI & Different AI tribes
- Part II Knowledge Representation & Reasoning

OUTLINE

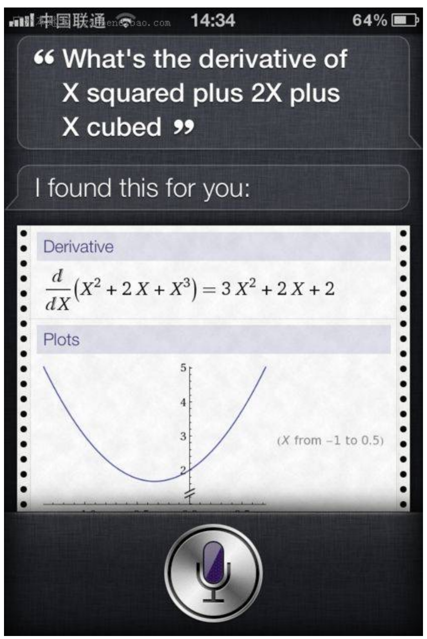
- *1.1 Brief Review*
- 1.2 Knowledge Representation & Reasoning

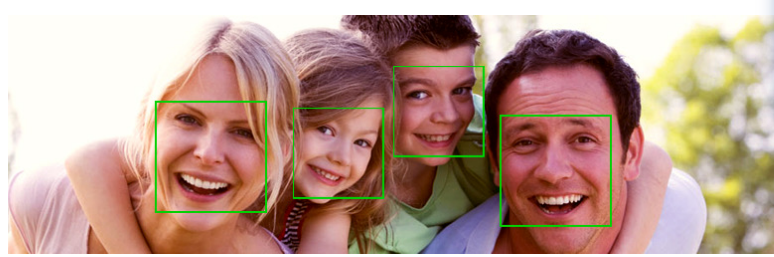
1.1 Brief Review

What Is Artificial Intelligence?

Actually,

artificial intelligence is intelligence exhibited by machines.





1.1 Brief Review

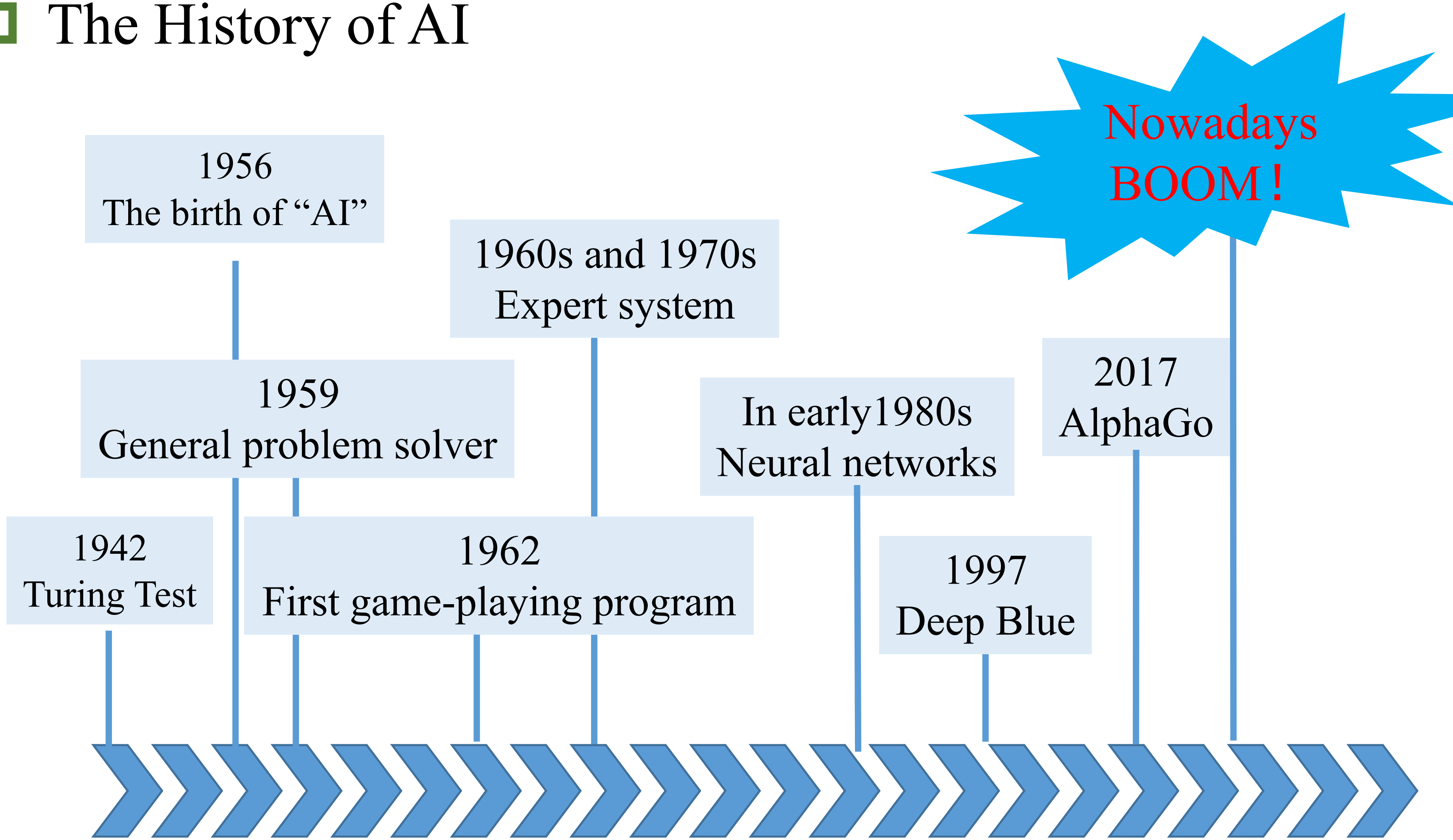
What Is Artificial Intelligence?

We agree with that:

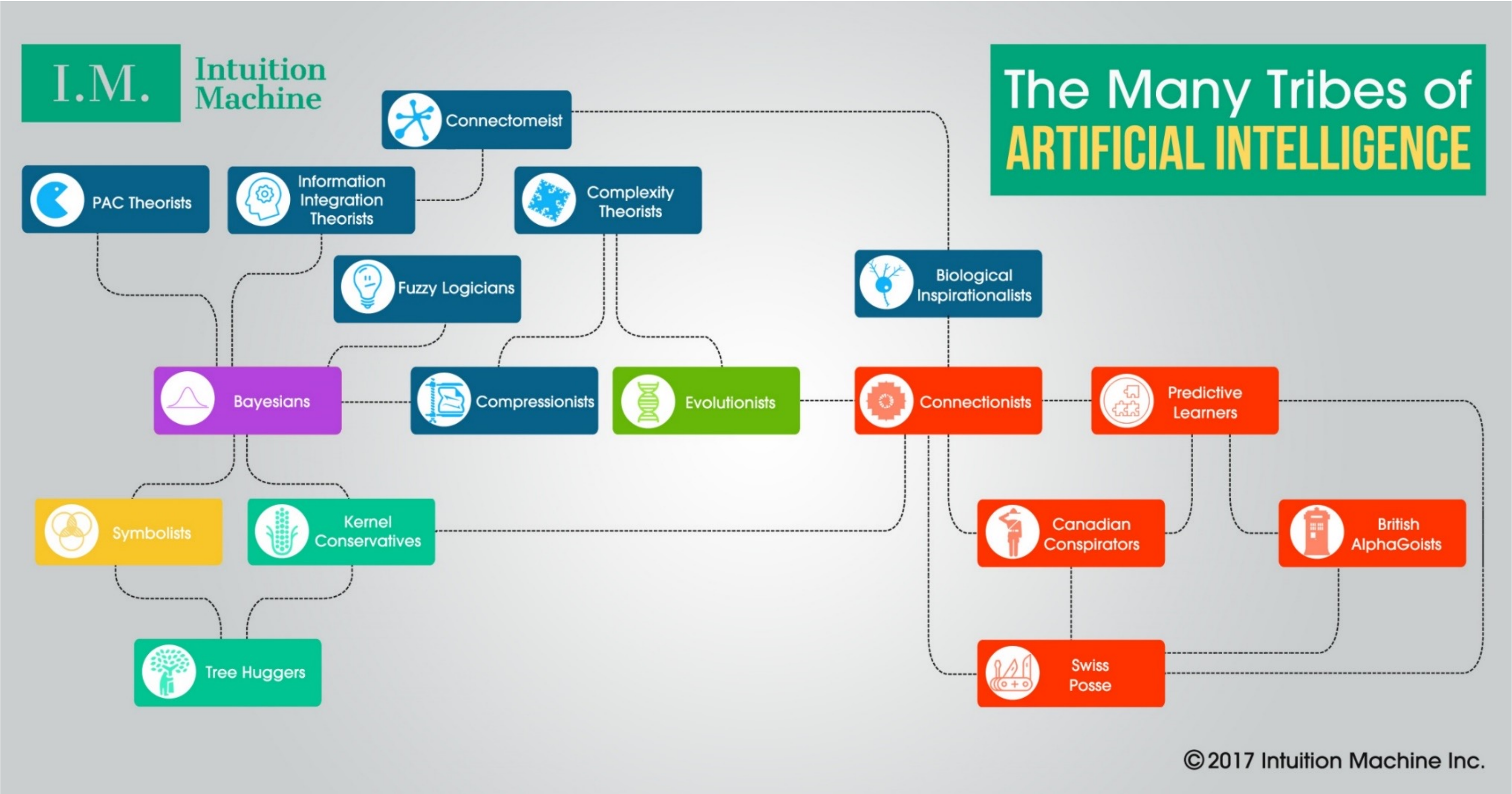
Intelligence is the ability to learn or understand or to deal with new or trying situations;
the ability to apply knowledge to manipulate one's environment or to think abstractly.

1.1 Brief Review

- The History of AI



1.2 Different Tribes of AI



1.2 Brief Review

- Traditional AI Methods

OUTLINE

- 1.1 Brief Review
- 1.2 *Knowledge Representation & Reasoning*

1.3 Knowledge Representation & Reasoning

- *What is knowledge representation?*
- Propositional Logic
- Predicate Logic
- Production-rule System
- Frame-Based System
- State Space System
- Knowledge graph

1.3 Knowledge Representation & Reasoning

Puzzle Time

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says “B is a knight.”
  - B says “The two of us are of opposite types.”

Example: What are the types of A and B?

How to represent the problem and solve it by computer?

Knowledge representation and reasoning

1.3 Knowledge Representation & Reasoning

Knowledge

- the information, understanding and skills accumulated in long-term life and social practice, scientific research and experiments.

- An information structure that links related information together.

- Knowledge reflects the relationship between things in the objective world.

Example:

1. The snow is in white. --- Facts
2. If you have a headache and a runny nose, you might have a cold. --- Rule

1.3 Knowledge Representation & Reasoning

Characteristic of Knowledge

- **Relative correctness**

Any knowledge is produced under certain conditions, and is correct under such conditions.

1+1=2 (decimal system)
1+1=10 (binary system)

1.3 Knowledge Representation & Reasoning

Characteristic of Knowledge

- Uncertainty： True, False, state between True and False
  - · Uncertainty caused by randomness
  - · Uncertainty caused by ambiguity
  - · Uncertainty caused by experience
  - · Uncertainty caused by incompleteness

Example:

1. If you have a headache and a runny nose, you might have a cold.
2. Li is very high.

1.3 Knowledge Representation & Reasoning

Characteristic of Knowledge

- Uncertainty caused by ambiguity

--- A vague concept

1.3 Knowledge Representation & Reasoning

- Characteristic of Knowledge

➢ Representability and Exploitability

Representability of knowledge: Knowledge can be expressed in appropriate forms, such as language, writing, graphics, neural networks, etc.

Exploitability of Knowledge : Knowledge can be utilized

1.3 Knowledge Representation & Reasoning

- Knowledge Representation
  - Formalize or model human knowledge.
  - a description of knowledge, or a set of conventions, a data structure that a computer can accept to describe knowledge.
  - Principles for selecting knowledge representation methods:
    - Fully express domain knowledge.
    - Conducive to the use of knowledge.
    - Easy to organize, maintain and manage
    - Easy to understand and implement.

1.3 Knowledge Representation & Reasoning

- What is knowledge representation?
- *Propositional Logic*
- Production-rule System
- Frame-Based System
- State Space System
- Knowledge graph

1.3 Knowledge Representation & Reasoning

What is propositional logic

Proposition:

A := The street is wet.

B := It is raining.

A **proposition** is a statement that is either true or false but not *both.*

- Atomic formulas are denoted by letters A, B, C, etc.
- Each atomic formula is assigned a truth value: true (1) or false (0).

1.3 Knowledge Representation & Reasoning

What is propositional logic

- A *proposition* is a declarative sentence that is either true or false.
- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Beijing is the capital of China.
  c) Hangzhou is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions.
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

1.3 Knowledge Representation & Reasoning

What is propositional logic

- It is possible to determine whether any given statement is a proposition by prefixing it with
- *It is true that . . .*
- and seeing whether the result **makes grammatical sense.**

  - What is the time?
  - 2 + 3 = 5
  - “Phone” has five letters.
  - 2 + 3 = 6
  - Oh dear!
  - I like AI class.

1.3 Knowledge Representation & Reasoning

What is propositional logic

- Have a try…
  - 您去电影院吗？ 不是命题
  - 2 + 3 = 5 命题
  - 看花去！ 不是命题
  - 这句话是谎言。 不是命题
  - X=2 不是命题
  - 两个奇数之和是奇数。 命题
  - 李白要么擅长写诗，要么擅长喝酒。 命题

1.3 Knowledge Representation & Reasoning

What is propositional logic

Proposition:

A := The street is wet.

B := It is raining.

> “ Propositional logic is not the study of truth, but of the relationship between the truth of one statement and that of another”
>
> ——Hedman 2004

We can connect the two propositions A and B:

If it is raining, the street is wet.

Written more formally

It is raining. → The street is wet.

A → B

1.3 Knowledge Representation & Reasoning

What is propositional logic

- Constructing Propositions
  - Propositional Variables: $p, q, r, s, \ldots$
  - The proposition that is always true is denoted by **T** and the proposition that is always false is denoted by **F**.
  - Compound Propositions: constructed from logical connectives and other propositions
    - Negation $\neg$ （否定联结词）
    - Conjunction $\wedge$ （合取联结词）
    - Disjunction $\vee$ （析取联结词）
    - Implication $\rightarrow$（蕴涵联结词）
    - Biconditional $\leftrightarrow$ （等价联结词）

1.3 Knowledge Representation & Reasoning

Syntax of logical connectives

- Conjunction : And $\wedge$
- Disjunction : Or $\vee$
- Negation : Not $\neg$
- Implication : Implies $\rightarrow$ (if… then…)
- Biconditional : Iff ↔️ (if and only if)

1.3 Knowledge Representation & Reasoning

AND（∧）

读作： “A并且B” “A与B”
称为：A与B的合取式
记作： $A \wedge B$

- The *conjunction* '*A AND B*', written $A \wedge B$, of two propositions is true when both A and B are true, false otherwise.

A	B	A ∧ B
t	t	t
t	f	f
f	t	f
f	f	f

Translation of sentences to propositions

A:= It's Monday.
B:= It's raining.

It's Monday and it's raining.
It's Monday but it's raining.
It's Monday. It's raining.

$\Big\}$ A ∧ B

1.3 Knowledge Representation & Reasoning

Translation of propositions to sentences

- In propositional logic :
- $A \wedge B$ and $B \wedge A$ should always have the same meaning.
- But…
- A:=She became sick .
- B:= she went to the doctor.

Logically the same!

- She became sick and she went to the doctor.
- and
- She went to the doctor and she became sick.

Different!

1.3 Knowledge Representation & Reasoning

OR　(∨)

- Also called *disjunction*.
- The disjunction "*A OR B*", written $A \vee B$, of two propositions is true when *A* or *B* (or both) are true, false otherwise.

读作：　"A或者B"
称为：A与B的析取式
记作：　$A \vee B$

A	B	A ∨ B
t	t	t
t	f	t
f	t	t
f	f	f

Translation of sentences to propositions

A:= It's Monday.
B:= It's raining.

It's Monday or it's raining.　A ∨ B

1.3 Knowledge Representation & Reasoning

NOT

读作："非A"
称为：A的否定式
记作：$\neg A$

- Also known as *negation*
- The negation "NOT A" of a proposition (or $\neg A$) is true when A is false and is false otherwise.
- $\neg A$ may be read that it is
- false that A.

A	¬A
t	f
f	t

Translation of sentences to propositions

$A :=$ AI is easy.

It is false that AI is easy.
It is not the case that AI is easy. $\neg$ A
AI is not easy.

1.3 Knowledge Representation & Reasoning

If . . . Then (→)

读作：“如果A则B”
称为：A与B的蕴涵式
记作：$A \rightarrow B$

- Also known as **implication**
- The implication “A IMPLIES B”, written $A \rightarrow B$, of two propositions is true when either A is false or B is true, and false otherwise.

A	B	A → B
t	t	t
t	f	f
f	t	t
f	f	t

A:= I study hard.
B:= I get rich.

$$\left.\begin{array}{l}\text{If I study hard then I get rich.}\\ \text{Whenever I study hard, I get rich.}\\ \text{That I study hard implies I get rich.}\\ \text{I get rich, if I study hard.}\end{array}\right\} A \rightarrow B$$

1.3 Knowledge Representation & Reasoning

If . . . Then ($\rightarrow$)

- In $p \rightarrow q$ there does not need to be any connection between the antecedent or the consequent.
- The "meaning" of $p \rightarrow q$ depends only on the truth values of p and q.

- These implications are perfectly fine, but would not be used in ordinary English.
  - "If the moon is made of green cheese, then I have more money than Bill Gates. "
  - "If the moon is made of green cheese then I'm on welfare."

1.3 Knowledge Representation & Reasoning

- Different Ways of Expressing $p \rightarrow q$

if p**, then** q	p **implies** q
if p**,** q	p **only if** q
q **unless** $\neg p$	q **when** p
q **if** p	
q **whenever** p	p **is sufficient for** q
q **follows from** p	q **is necessary for** p

a necessary condition for p **is** q

a sufficient condition for q **is** p

1.3 Knowledge Representation & Reasoning

Converse, Contrapositive, and Inverse

- From $p \to q$ we can form new conditional statements .
  - $q \to p$ is the **converse** of $p \to q$
  - $\neg q \to \neg p$ is the **contrapositive** of $p \to q$
  - $\neg p \to \neg q$ is the **inverse** of $p \to q$

Example: Find the converse, inverse, and contrapositive of "It's raining is a sufficient condition for my not going to town."

Solution:

converse: ?

inverse: ?

contrapositive: ?

1.3 Knowledge Representation & Reasoning

Converse, Contrapositive, and Inverse

- From $p \to q$ we can form new conditional statements .
  - $q \to p$ is the **converse** of $p \to q$
  - $\neg q \to \neg p$ is the **contrapositive** of $p \to q$
  - $\neg p \to \neg q$ is the **inverse** of $p \to q$

Example: Find the converse, inverse, and contrapositive of "It's raining is a sufficient condition for my not going to town."

Solution: → "If it is raining, I do not go town."

converse: If I do not go to town, then it is raining.

inverse: If it is not raining, then I will go to town.

contrapositive: If I go to town, then it is not raining.

1.3 Knowledge Representation & Reasoning

Biconditional

读作： “A当且仅当B”
称为：A与B的等价式
记作： $A \leftrightarrow B$

- Also known as iff or the biconditional.

The biconditional, written as $A \leftrightarrow B$, of two propositions is true when both A and B are true or when both A and $B$are false, and false otherwise.

A	B	A ↔ B
t	t	t
t	f	f
f	t	f
f	f	t

If p denotes “I am at home.” and q denotes “It is raining.” then $p \leftrightarrow q$ denotes “I am at home if and only if it is raining.”

1.3 Knowledge Representation & Reasoning

Semantics

- Example:

A := The street is wet.

B := It is raining.

If A is true, and B is true, then A ∧ B is true.

Truth table

A	B	A ∧ B
t	t	t
t	f	f
f	t	f
f	f	f

Interpretation

-A line in the truth table.

interpretation

A	B	¬ A	A ∧ B	A ∨ B	A → B	A ↔ B
t	t	f	t	t	t	t
t	f	f	f	t	f	f
f	t	t	f	t	t	f
f	f	t	f	f	t	t

1.3 Knowledge Representation & Reasoning

Tautologies, Contradictions, and Contingencies

- A tautology (永真式) is a proposition which is always true.
  - Example: $p \vee \neg p$
- A *contradiction* (矛盾式) is a proposition which is always false.
  - Example: $p \wedge \neg p$
- A *contingency* is a proposition which is neither a tautology nor a contradiction, such as p

1.3 Knowledge Representation & Reasoning

Logical Equivalences

- Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.
- We write this as $p \Leftrightarrow q$ or as $p \equiv q$ where p and q are compound propositions.
- Two compound propositions p and q are equivalent if and only if the columns in a truth table giving their truth values agree.
- This truth table shows that $\neg p \vee q$ is equivalent to $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

1.3 Knowledge Representation & Reasoning

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan

1806-1871

This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T					
T	F					
F	T					
F	F					

1.3 Knowledge Representation & Reasoning

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

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This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

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p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T		
T	F	F	T	T		
F	T	T	F	T		
F	F	T	T	F		

1.3 Knowledge Representation & Reasoning

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

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This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	
T	F	F	T	T	F	
F	T	T	F	T	F	
F	F	T	T	F	T	

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Augustus De Morgan

1806-1871

This truth table shows that De Morgan's Second Law holds.

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

1.3 Knowledge Representation & Reasoning

Key Logical Equivalences

- Identity Laws: $p \wedge T \equiv p$, $p \vee F \equiv p$
- Domination Laws: $p \vee T \equiv T$, $p \wedge F \equiv F$
- Idempotent laws: $p \vee p \equiv p$, $p \wedge p \equiv p$
- Double Negation Law: $\neg(\neg p) \equiv p$
- Negation Laws: $p \vee \neg p \equiv T$, $p \wedge \neg p \equiv F$

1.3 Knowledge Representation & Reasoning

Key Logical Equivalences

- Commutative Laws: $p \vee q \equiv q \vee p\ ,\quad p \wedge q \equiv q \wedge p$
- Associative Laws:
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
- Distributive Laws:
$$(p \vee (q \wedge r)) \equiv (p \vee q) \wedge (p \vee r)$$
$$(p \wedge (q \vee r)) \equiv (p \wedge q) \vee (p \wedge r)$$
- Absorption Laws: $p \vee (p \wedge q) \equiv p \quad p \wedge (p \vee q) \equiv p$

1.3 Knowledge Representation & Reasoning

Logical Equivalences

- *The operations* $\wedge, \vee$ *are commutative and associative, and the*
- *following equivalences are generally valid*:
- $\neg A \vee B \equiv A \rightarrow B$ (implication)
- $A \rightarrow B \equiv \neg B \rightarrow \neg A$ (contraposition)
- $(A \rightarrow B) \wedge (B \rightarrow A) \equiv (A \leftrightarrow B)$ (equivalence)
- $\neg (A \wedge B) \equiv \neg A \vee \neg B$ (De Morgan's law)
- $\neg (A \vee B) \equiv \neg A \wedge \neg B$
- $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ (distributive law)
- $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
- $A \vee \neg A \equiv t$ (tautology)
- $A \wedge \neg A \equiv f$ (contradiction)
- $A \vee f \equiv A$
- $A \vee t \equiv t$
- $A \wedge f \equiv f$
- $A \wedge t \equiv A$

1.3 Knowledge Representation & Reasoning

Logical Equivalences

- *The operations* $\wedge, \vee$ *are commutative and associative, and the*
- *following equivalences are generally valid*:
- $\neg A \vee B \equiv A \rightarrow B$ (implication)
- $A \rightarrow B \equiv \neg B \rightarrow \neg A$ (contraposition)

A	B	¬ A	¬A∨ B	A → B
t	t	f	t	t
t	f	f	f	f
f	t	t	t	t
f	f	t	t	t

1.3 Knowledge Representation & Reasoning

- Equivalence Proofs

Example: Show that $\neg(p \vee (\neg p \wedge q))$ is logically equivalent to $\neg p \wedge \neg q$

Solution:

1.3 Knowledge Representation & Reasoning

Equivalence Proofs

Example: Show that $\neg(p \vee (\neg p \wedge q))$ is logically equivalent to $\neg p \wedge \neg q$

Solution:

$$
\begin{array}{llll}
\neg(p \vee (\neg p \wedge q)) & \equiv & \neg p \wedge \neg(\neg p \wedge q) & \text{by the second De Morgan law} \\
 & \equiv & \neg p \wedge [\neg(\neg p) \vee \neg q] & \text{by the first De Morgan law} \\
 & \equiv & \neg p \wedge (p \vee \neg q) & \text{by the double negation law} \\
 & \equiv & (\neg p \wedge p) \vee (\neg p \wedge \neg q) & \text{by the second distributive law} \\
 & \equiv & F \vee (\neg p \wedge \neg q) & \text{because } \neg p \wedge p \equiv F \\
 & \equiv & (\neg p \wedge \neg q) \vee F & \text{by the commutative law} \\
 & & & \text{for disjunction} \\
 & \equiv & (\neg p \wedge \neg q) & \text{by the identity law for } \mathbf{F}
\end{array}
$$

1.3 Knowledge Representation & Reasoning

- Equivalence Proofs

Example: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution:

Equivalence Proofs

Example: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution:

$$
\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{by truth table for } \rightarrow \\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by the first De Morgan law} \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{by associative and} \\
& && \text{commutative laws} \\
& && \text{laws for disjunction} \\
&\equiv T \vee T && \text{by truth tables} \\
&\equiv T && \text{by the domination law}
\end{aligned}
$$

1.3 Knowledge Representation & Reasoning

- Now, we have learned…
  - Three basic elements in proposition logic: propositions, operations, and the truth values.
  - Logical equivalences

1.3 Knowledge Representation & Reasoning

- Applications
  - 1. Translate English Sentences
  - 2. System Specifications
  - 3. Logic Puzzles
  - 4. Logic Circuit

1.3 Knowledge Representation & Reasoning

Example

Problem: Translate the following sentence into propositional logic:
"You can access the Internet from campus if you are a computer science major or you are not a freshman."

Atomic propositions:

- A:= You can access the Internet from campus.
- B:= You are a computer science major .
- C:= You are a freshman.

$$(B \vee \neg C) \rightarrow A$$

1.3 Knowledge Representation & Reasoning

Consistent System Specifications

- Definition: A list of propositions is consistent if it is possible to assign truth values to the proposition variables so that each proposition in the list is true.

1.3 Knowledge Representation & Reasoning

- Consistent System Specifications

Example: Is this list of propositions consistent?

- "The diagnostic message is stored in the buffer or it is retrans mitted."
- "The diagnostic message is not stored in the buffer."
- "If the diagnostic message is stored in the buffer, then it is ret ransmitted."

1.3 Knowledge Representation & Reasoning

Consistent System Specifications

Example: Is this list of propositions consistent?

- "The diagnostic message is stored in the buffer or it is retransm itted."
- "The diagnostic message is not stored in the buffer."
- "If the diagnostic message is stored in the buffer, then it is retra nsmitted."

P:= "The diagnostic message is stored in the buffer."
Q:= "The diagnostic message is retransmitted"

$$P \vee Q$$

1.3 Knowledge Representation & Reasoning

Consistent System Specifications

Example: Is this list of propositions consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted."
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P:= "The diagnostic message is stored in the buffer."
Q:= "The diagnostic message is retransmitted"

$$\neg P$$

1.3 Knowledge Representation & Reasoning

Consistent System Specifications

Example: Is this list of propositions consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted."
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P:= "The diagnostic message is stored in the buffer."
Q:= "The diagnostic message is retransmitted"

$$P \rightarrow Q$$

1.3 Knowledge Representation & Reasoning

Consistent System Specifications

Example: Is this list of propositions consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted." $P \vee Q$
- "The diagnostic message is not stored in the buffer." $\neg P$
- "If the diagnostic message is stored in the buffer, then it is retransmitted." $P \rightarrow Q$

P	Q	P ∨ Q	¬ P	P → Q
t	t			
t	f			
f	t			
f	f			

1.3 Knowledge Representation & Reasoning

Consistent System Specifications

Example: Is this list of propositions consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted."
- "The diagnostic message is not stored in the buffer."
- "If the diagnostic message is stored in the buffer, then it is retransmitted."

P	Q	$P \lor Q$	$\neg P$	$P \rightarrow Q$
t	t	t	f	t
t	f	t	f	f
f	t	t	t	t
f	f	f	t	t

When P is false and Q is true all three statements are true. So the list of propositions is consistent.

1.3 Knowledge Representation & Reasoning

Logic Puzzles

Knights: t
Knaves: f

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says "B is a knight."
  - B says "The two of us are of opposite types."

Example: What are the types of A and B?

1.3 Knowledge Representation & Reasoning

Logic Puzzles

- **Solution:** Let p and q be the statements that A is a knight and B is a knight, respectively. So, then $\neg p$ represents the proposition that A is a knave and $\neg q$ that B is a knave.
  - If A is a knight, then p is true. Since knights tell the truth, q must also be true. Then $(p \wedge \neg q) \vee (\neg p \wedge q)$ would have to be true, but it is not. So, A is not a knight and therefore $\neg p$ must be true.
  - If A is a knave, then B must not be a knight since knaves always lie. So, then both $\neg p$ and $\neg q$ hold since both are knaves.

Homework-1

Logic Puzzles

Knights: t
Knaves: f

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says "At least one of us is a knave."
  - B says nothing.

Example: What are the types of A and B?